Midterm 1: Math 321
February 23, 2023, 9:30-10:45am

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
Total	120	

Instructions

- Print your name below and sign the statement. Exams which lack these will not be graded.
- Be prepared to show your ID to the proctor when turning in the exam.
- Make sure that your work is shown in a neat and organized manner. If you need more space to answer a problem, use the back of a page, making a note where the additional work can be found.
- You are allowed both sides of a single 8.5” x 11” sheet of paper for notes. Hand your note sheet in with your exam.

Please sign and date the following statement:
I have read the instructions and understand them. I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Printed Name Signature Date

1. Find all solutions, if any, to the linear system

$$\begin{aligned} x_2 + 2x_3 &= 1 \\ x_1 + 3x_2 + 2x_2 &= 2 \\ x_1 + x_2 - 2x_3 &= 1 \end{aligned}$$

Show your work.

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(a) Transform A to reduced echelon form.

(b) Do the columns of A span $\mathbb{R}^3$? Explain your answer.

(c) Find all solutions to the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

3. Given a vector $\mathbf{v}$ in $\mathbb{R}^3$, denote its entries as

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}.$$

Also define the following vectors in $\mathbb{R}^3$

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \ \mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}.$$

Show that if $2v_1 + v_2 + v_3 = 0$, then $\mathbf{v}$ is in the span of $\mathbf{u}_1, \mathbf{u}_2$.

4. Determine all values of h for which the following vectors are linearly independent. Explain your answer:

$$\begin{bmatrix} 1 \\ -2 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ h \\ 2 \end{bmatrix}.$$

5. Using minimal calculations, explain why the following sets of vectors are linearly *dependent*.

(a)

$$\mathbf{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 29 \\ 32 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 172 \\ 173 \end{bmatrix}$$

(b)

$$\mathbf{u} = \begin{bmatrix} 0 \\ 10 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 55 \\ 40 \\ 2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 55 \\ 50 \\ 4 \end{bmatrix}$$

Hint for (b): What is $\mathbf{u} + \mathbf{v}$?

6. Let $T(\mathbf{x}) = A\mathbf{x}$ for the given matrix A. Determine if T is one-to-one. Justify your answer.

(a)

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 365 & 97 \\ 0 & 0 & \frac{1}{597} \end{bmatrix}$$

Note: Part (b) can be solved without messy calculations.