# MATH 121: EXTRA PRACTICE FOR TEST 2

**Disclaimer:** Any material covered in class and/or assigned for homework is a fair game for the exam.

## 1 Linear Functions

1. Consider the functions $f(x) = 3x + 5$ and $g(x) = -2x + 15$.
   (a) Solve $f(x) = 0$.
   (b) Solve $f(x) = g(x)$.
   (c) Solve $f(x) < 0$.
   (d) Solve $f(x) \geq g(x)$.
   (e) Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution to the equation $f(x) = g(x)$.

2. The cost $C$, in dollars, of renting a moving truck for a day is given by the function $C(x) = 0.25x + 35$, where $x$ is the number of miles driven.
   (a) What is the cost if you drive $x = 40$ miles?
   (b) If the cost of renting the moving truck is $80, how many miles did you drive?
   (c) Suppose that you want the cost to be no more than $100. What is the maximum number of miles that you can drive?

3. A phone company offers a domestic long distance package by charging $5 plus $0.05 per minute.
   (a) Write a linear function that relates the cost $C$, in dollars, of renting the truck to the number $x$ of miles driven.
   (b) What is the cost of renting the truck if the truck is driven 110 miles? 230 miles?

## 2 Quadratic Functions

4. Recall that the general form of a quadratic function is $f(x) = ax^2 + bx + c$, where $a$, $b$, $c$ are real numbers and $a \neq 0$. Also, the standard form of a quadratic function is $f(x) = a(x - h)^2 + k$, where $h$ and $k$ are real numbers.
   (a) What is the domain of any quadratic function?
   (b) What is the name of the graph of any quadratic function?
   (c) If $a > 0$, does the parabola open up or down?
   (d) If $a > 0$, does the parabola have a maximum or a minimum?
   (e) Where does the maximum/minimum of a quadratic function occur?
   (f) What is the vertex of the parabola (written in general form, in terms of $a$, $b$ and $c$)?
   (g) Make sure that you are able to write any quadratic function in standard form.
   (h) What is the axis of symmetry of a parabola?
   (i) How do you find the $x-$intercepts?
   (j) What is an alternative way of finding the location of the vertex of a parabola that has two $x$-intercepts (take advantage of symmetry)?
   (k) What is the $y-$intercept of a parabola?
   (l) True or False: A quadratic function is a polynomial.

5. Consider the quadratic functions given below. For each one:

   - Determine if the graph will be a parabola open up or down.
   - Find any $x-$ intercepts.
   - Find the $y-$intercept.
   - Determine the vertex of the parabola.
   - Find the axis of symmetry.
   - Write the standard form of the quadratic function.
   - Find the location and the value of the maximum/minimum of the function.
   - Find the intervals where the function is increasing and where it is decreasing.
   - What are the domain and the range of the quadratic equation?
   - Sketch the graph of the function. Make sure to label the axis, the vertex and the intercepts.

   (a) $f(x) = -x^2 + 6x - 9$
   (b) $k(x) = 2x^2 - 4x + 1$
   (c) $g(x) = -2x^2 + 4$
   (d) $h(x) = 6 - x - x^2$

6. Consider $f(x) = -x^2 - 2x + 1$. Write it in standard form and sketch the graph by starting with the graph of $f(x) = x^2$ and using transformations (shifting, stretching/compressing, and/or reflecting).

7. Find the $x-$coordinate of the vertex of the quadratic function $f(x) = 3(x-2)(x+4)$, by taking advantage of the fact that parabolas have axis of symmetry. What is the $y-$ coordinate of the vertex?

8. David is the manager of a tire repair shop. He found that the relationship between the price, $p$, to fix a flat, and the number of flats people brought in, $x$, is $p = -\frac{1}{4}x + 40$.

   (a) Find the revenue as a function of $x$, the number of flats fixed.
   (b) What is the domain of the function (within the context)?
   (c) What is the revenue if he fixes 40 flats?
   (d) How many flats should he fix to make the maximum revenue?
   (e) What price should David charge to maximize his revenue?
   (f) What is the maximum revenue that David can make fixing flats?

9. The height above ground of a projectile thrown straight upward from a building is given by

$$h(t) = -16t^2 + 32t + 48,$$

   where $h$ is measured in feet and $t$ in seconds.

   (a) How high above ground is the projectile when it is launched?
   (b) When will it reach 48 ft again?
   (c) What is the maximum height it will reach?
   (d) How high is it after 1.5 seconds?
   (e) When will it land?

10. David has 400 meters of fencing and wants to enclose a rectangular area.

    (a) Express the area $A$ of the rectangle as a function of the width $w$ of the rectangle.
    (b) For what value of $w$ is the area largest?
    (c) What is the maximum area?

# 3 Polynomial Functions

Recall that a polynomial of degree $n$ is a function of the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0,$$

where $a_n, a_{n-1}, ..., a_1, a_0$ are real numbers and $n$ is a nonnegative integer. The coefficient in front of $x^n$, $a_n$, is called a **leading coefficient**.

(a) What is the domain of any polynomial function?

(b) What is a **zero** of a polynomial?

(c) Describe what can happen with the graph of the function at the real zeros.

(d) What can you say about the multiplicity of a zero if the graph of the polynomial crosses the $x-$axis at that zero?

(e) What do you understand by 'end behavior'?

(f) What is a turning point?

(g) How many turning points can a polynomial of degree $n$ have?

(h) If $f(x) < 0$ in some interval, what can you say about the graph of $f(x)$ in that interval?

**TO SKETCH THE GRAPH OF A POLYNOMIAL FUNCTION:**

(a) Determine the degree of the polynomial.

(b) Determine the sign of the leading coefficient.

(c) Based on the degree and the sign of the leading coefficient, determine the end behavior of the graph.

(d) Make sure that the function is in completely factored form.

(e) Find the zeros.

(f) Find the multiplicities, $m$, of each of the zeros.

(g) Depending whether $m$ is odd/even, determine if the graph will cross/touch the $x$-axis at each real zero.

(h) Find the $y$-intercept.

(i) Determine the maximum number of turning points the graph can have.

(j) If you need additional information in order to plot the graph: plot the zeros/x-intercepts on a number line and pick a point in each interval to determine if the graph of the function is above/below the $x-$axis in that interval.

(k) Plot the information obtained in the above steps and use a smooth, continuous curve to plot the graph.

11. Follow the steps given above to sketch the graph of each of the given polynomial functions:

(a) $f(x) = (x-4)(x+2)^2(x-2)$

(b) $g(x) = -4x^3 + 4x$

(c) $p(x) = x(x-1)^2(x+3)(x+1)$

(d) $h(x) = (x^2-25)(x+5)$

(e) $q(x) = -2x^4 + 4x^2$

(f) $r(x) = \frac{1}{2}(x^2+1)(x^2-3)$

12. Form a polynomial with zeros at -2, 0, and 7 and multiplicities 2, 3, 1 respectively, whose leading coefficient -3. What is the degree of the resulting polynomial?

## 4 Rational Functions

Recall that a rational function is a function of the form

$$R(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials and $Q(x)$ is not the zero polynomial.

(a) What is the domain of a rational function?

(b) What do the horizontal/slant asymptote tell you?

(c) What does a vertical asymptote tell you?

(d) When does the graph of a rational function have horizontal/slant/vertical asymptotes?

(e) What can you conclude about the graph of the rational function $R(x)$ if as $x \to \infty$, $R(x) \to 3$.

(f) What can you conclude about the graph of the rational function $R(x)$ if as $x \to 3$, $R(x) \to \infty$.

(g) When does a rational function have horizontal/slant asymptote?

(h) When does $R(x)$ have vertical asymptotes?

(i) When does $R(x)$ have hole(s) in the graph?

(j) Can $R(x)$ have both a horizontal and a slant asymptote? Can it have more than one horizontal/slant asymptotes?

(k) Can $R(x)$ have more than one vertical asymptotes?

(l) Can the graph of $R(x)$ cross any of the asymptotes?

13. Find all the vertical, horizontal and slant asymptotes, and any holes in the graphs of the given rational functions:

(a) $f(x) = \dfrac{2x^2 + 7x + 3}{x + 1}$

(b) $f(x) = \dfrac{2x^2 + 1000000 + x}{3x - x^2}$

(c) $f(x) = \dfrac{x + 2}{x^2 - 4}$

(d) $f(x) = \dfrac{x^2 - 4}{x + 2}$

(e) $f(x) = \dfrac{10000x^2 + 3x - 1}{x^3}$

(f) $R(x) = \dfrac{3x^4 + 3}{-7x + 2x^4}$

**TO SKETCH THE GRAPH OF A RATIONAL FUNCTION:**

(a) Find the domain.

(b) Find the x-intercepts (if the numerator and the denominator do not have common factors then the x-intercepts of the rational function are the zeros of the numerator.)

(c) Find the $y-$intercept.

(d) Find the asymptotes and the holes, if any.

(e) Check if the graph of the function crosses the horizontal/slant asymptote.

(f) Plot the zeros and the points where the function is undefined on a number line. Pick a number in each interval to determine if the graph of the rational function is above or below the $x-$axis.

(g) Plot the asymptotes, the intercepts and use the information from the previous steps to obtain the graph of $R(x)$.

14. Following the steps given above, sketch the graph of the given rational functions:

(a) $f(x) = \dfrac{2x^2 + 2x - 12}{x^2 - x - 6}$

(b) $f(x) = \dfrac{x^3}{x^2 - 4}$

(c) $f(x) = \dfrac{3x^3}{(x-1)^2}$

(d) $f(x) = \dfrac{(x-1)^2}{x^2 - 1}$

(e) $f(x) = \dfrac{x^4}{x^2 - 9}$

(f) $f(x) = \dfrac{x^2 - 9}{x^2 - 2x - 3}$

(g) $f(x) = \dfrac{(x+2)(x^2 - 2x + 1)}{x^2 + 5x + 6}$

15. Make up a function that fits the listed conditions:

(a) A function whose domain is all reals except 3.

(b) A function whose range is everything except 0.

(c) A function whose graph has vertical asymptotes at 2 and 5 and a horizontal asymptote at 1.

(d) A function whose graph has a horizontal asymptote at $y = 0$ and no vertical asymptotes.

## 5 Inequalities

16. Solve the given inequalities. Represent your solution, using interval notation.

(a) $x^2 < x$

(b) $x^2 + 1 \leq 0$

(c) $x^2 + 2x \geq -1$

(d) $|x + 5| - 5 < 7$

(e) $2|7 - x| + 1 > 4$

(f) $x^2 \leq 3 - 2x$

(g) $5x + 4 \geq 3x^2$

(h) $5x + 4 < 3x^2$

(i) $\left|\dfrac{11}{4}x - 3\right| \geq \dfrac{15}{4}$

(j) $\dfrac{(x-2)(x-1)}{x+3} \geq 0$

(k) $\dfrac{x+1}{x(x+3)} \leq 0$

(l) $\dfrac{6}{x+3} \geq 0$

(m) $\dfrac{6}{x+3} \geq 1$

(n) $\dfrac{6}{x+3} < 1$

(o) $\dfrac{x^2 - 8x + 12}{x^2 - 16} < 0$

(p) $\dfrac{6}{x^2 + 3} \geq 0$

(q) $\dfrac{3x - 1}{x^2 + 1} \geq 1$

(r) $\dfrac{2x + 17}{x + 1} \geq x + 5$