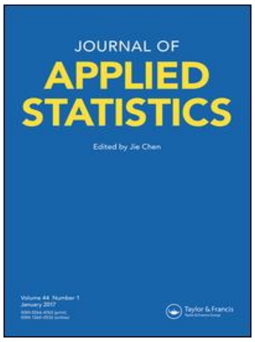

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NOTE

Identification of technical analysis patterns with smoothing splines for bitcoin prices

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ABSTRACT

This research studies automatic price pattern search procedure for bitcoin cryptocurrency based on 1-min price data. To achieve this, search algorithm is proposed based on nonparametric regression method of smoothing splines. We investigate some well-known technical analysis patterns and construct algorithmic trading strategy to evaluate the effectiveness of the patterns. We found that method of smoothing splines for identifying the technical analysis patterns and that strategies based on certain technical analysis patterns yield returns that significantly exceed results of unconditional trading strategies.

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KEYWORDS

Algorithmic trading strategy; bitcoin cryptocurrency; smoothing splines; technical analysis patterns; pattern recognition

1. Introduction

Bitcoin is a cryptocurrency. As the world's first decentralized (designed to work without a central bank or single administrator) digital payment method [1,10], bitcoin has been gradually recognized and accepted as a medium of exchange and a store of value. There are numerous revolutionary innovations of bitcoin and cryptocurrencies. One of them is blockchain [4], originally the public ledger that records bitcoin transactions, that has inspired its inventive applications as decentralized transaction ledger for alternative cryptocurrencies and other forms of records. To acquire bitcoins, one needs to either mine for them or receive them as a medium of exchange in a trade. Bitcoin's value has increased dramatically since its inception in 2009, from $0.003 in March 2009 to approximately $9000.00 in May 2018. Trading of bitcoin for fiat currencies and other cryptocurrencies has become very popular among amateur and professional investors.

Technical analysis is an approach to investment that relies upon the idea that analysis of trends in financial markets can be used to maximize profits through strategic buy and sell decisions. Statistical tools such as nonparametric kernel regression have been adopted to construct systematic and automatic approach to technical pattern recognition. For example, Lo et al. [8] and Lo and MacKinlay [7] studied U.S. equity market and shown existence of technical analysis patterns; Dawson and Steeley [3] worked on the UK stock market and found a weak pattern compared to that of NYSE and NASDAQ equity market; Wang et al.

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[13] investigated Chinese stock market. Unfortunately, many technical patterns mentioned in Lo et al. [8] are not significant in Chinese stock market. Wang and Zhang [12] proposed a complete data-driven technical analysis algorithm with the application of nonparametric local linear estimator and found that algorithms are informative.

The cryptocurrency market, however, is different from traditional markets due to a unique property which allows trading around the clock 24 h every day without any days off. This means traders are not confined to a finite time frame like traditional markets. Furthermore, individuals and business can trade globally on the same market at any time. Trading algorithms, therefore, are needed and become more and more popular, as an increasing share of trading is done by trading robots. In this research, we introduce a nonparametric regression tool smoothing splines to the analysis of bitcoin market. We attempt to test if some technical analysis patterns are still useful and profitable in the brand-new market: bitcoin market.

This article is organized as follows. Section 2 gives a background review of smoothing splines. Section 3 describes our proposed methodology on technical analysis and evaluation procedure. Section 4 gives the data analysis report. Finally, we give a conclusion in Section 5.

2. Background

2.1. Smoothing splines

Consider the general nonparametric regression model

$$y_i = \mu(t_i) + \varepsilon_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

where $\{\varepsilon_i\}_{i=1}^n$ is a sequence of independent, identically distributed random variables with $\mathrm{E}(\varepsilon_i) = 0$ and $\mathrm{E}(\varepsilon_i^2) = \sigma^2$, $\mu(\cdot)$ is an unknown smooth regression curve in the second order Sobolev space $W_2^2[0, 1]$ (μ and μ' are absolutely continuous and μ'' is a Lebesgue integrable function) to be estimated. Without loss of generality, we take $t_i \in [0, 1], i = 1, 2, \ldots, n$ and for simplicity we assume that $0 < t_1 < \cdots < t_n < 1$.

A natural cubic spline is a smooth piecewise cubic polynomial under certain boundary constraints, which can be obtained by minimizing

$$\frac{1}{n}\sum_{i=1}^{n}(y_i - f(t_i))^2 + \lambda \int_0^1 (f''(t))^2 \,\mathrm{d}t \tag{2}$$

over $f \in W_2^2[0, 1]$, where $\lambda > 0$ is the smoothing parameter that controls the tradeoff between smoothness and goodness-of-fit. The first term in (2) is the residual sum of squares which is a standard measure of goodness-of-fit to the data. The second term in (2) is a natural measure of curvature of the function. The smoothing spline estimator can be derived as follows [5]:

$$\hat{\boldsymbol{\mu}}_\lambda = (\hat{\mu}_\lambda(t_1), \ldots, \hat{\mu}_\lambda(t_n))^{\mathrm{T}} = \mathbf{S}_\lambda \mathbf{y}, \tag{3}$$

where $\mathbf{S}_\lambda = \mathbf{X}(\mathbf{X}^{\mathrm{T}}\mathbf{X} + n\lambda\Omega)^{-1}\mathbf{X}^{\mathrm{T}}$, $\mathbf{X} = \{x_j(t_i)\}_{i,j=1,\ldots,n}$, $\Omega = \{\int_0^1 x_i(t)x_j(t)\,\mathrm{d}t\}_{i,j=1,\ldots,n}$, $\mathbf{y} = (y_1, \ldots, y_n)$ and $x_1, x_2, \ldots, x_n$ is a basis for the set of natural cubic splines with knots at $t_1, \ldots, t_n$.

2.2. Selection of λ

The remaining problem is to choose the smoothing parameter λ. Two commonly used techniques for smoothing parameter selection are the cross validation (CV) method and the generalized cross validation (GCV) method [2]. GCV had a variety of statistical applications [11] and is nearly an unbiased estimator of prediction risk, while CV is biased.

The GCV criterion is defined as

$$GCV(\lambda) = \frac{\frac{1}{n}\sum_{i=1}^{n}(y_i - \mu_\lambda(t_i))^2}{\left(\frac{1}{n}\mathrm{tr}(I - S_\lambda)\right)^2}, \tag{4}$$

where $\mathrm{tr}(\cdot)$ denotes the trace of the matrix and λ is chosen to minimize (4). Zhang [15] showed that the GCV criterion is more likely to derive the estimate of λ with smaller variance for smoothing splines.

3. Dataset and methodology

In this section, we describe dataset used in this research and propose technical analysis procedure. Six different technical analysis patterns are considered and smoothing splines are used to smooth out the noise of movements, which enables identification of patterns on the smoothed curve. We also proposed a method to evaluate the effectiveness of our strategic trading algorithm.

3.1. Dataset

The data source is Global Digital Assets Exchange (GDAX). We wrote a program in Python to collect data on bitcoin prices in period of 5th February 2018 to 6th March 2018, which constitutes a total of 43,199 observations that are recorded every minute. The data are installed in Structured Query Language (SQL) local server through PHPMyAdmin. We then read the data into R and perform all the computations and simulations in R. At the time of writing this project, 1 min data are the finest price data that is available on GDAX. The dataset contains information on volume and low, high, open and close prices of bitcoin in terms of USD on GDAX for the each minute. This dataset has one missing entry of 1 min in the data table, an error which was present in the database GDAX. This source of error can be considered insignificant, as it does not contribute to any significant inaccuracy in the results, given our big dataset.

3.2. Pattern rules

The patterns considered in this research had been investigated on stock market data by other authors such as Lo et al. [8], Lo and MacKinlay [7] and Wang and Zhang [12]. Suppose that there are T trading time points (1 min data are considered as one point) with the sequence of closing price $\{P_i\}$ and trading volume $\{V_i\}$, $i = 1, 2, \ldots, T$. To identify a pattern, a window of every 35 min data is fitted by smoothing splines. The smoothed data provide the local minimum and maximum values needed to identify the price patterns. We denote these local extrema as $E_1, E_2, \ldots, E_k$ corresponding to $t_1^*, t_2^*, \ldots, t_k^*$ on the occurrences of the trading time point.

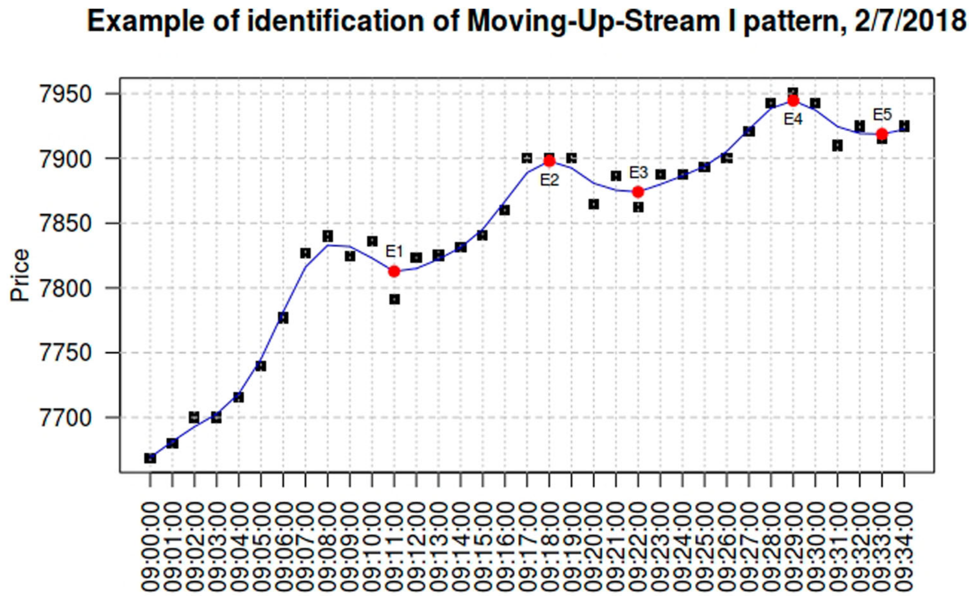

Figure 1. Price pattern: Moving-Up-Stream.

In particular, we denote E_1, E_2, E_3, E_4, E_5 the five local extreme closing price points on the smoothed curve, with E_5 being the closest extreme point to the end of time subinterval of model fit. The interpretations of these points as either maximum or minimum points on the smoothed curve are given by the particular technical analysis patterns. The trading volumes V_1, V_2, V_3, V_4 are corresponding to the extreme price points E_1, E_2, E_3, E_4. We only search for patterns during the time subintervals when at least five extreme points are identified.

The quantitative interpretations of pattern rules of Head-And-Shoulders, Broadening, Triangle, Rectangle and Double Tops and Bottoms originally appeared in Lo et al. [8] and Lo and MacKinlay [7], and Moving-Up-Stream originally appeared in Wang and Zhang [12]. The patterns are adapted to be implemented within 35 min time subintervals. Below are the quantitative interpretations of the pattern rules. Figure 1 illustrates an example of the price pattern shape of the Moving-Up-Stream I and II patterns by using 35 min data on February 7th 2018. It should be noted that the presence of patterns is investigated on smoothing splines model fits, not on the price data itself.

- Head-And-Shoulders: E_1 is a maximum, $E_3 > E_1, E_3 > E_5$, E_1 and E_5 are within 1.5% of their average, E_2 and E_4 are within 1.5% of their average. Pattern gives sell signal, and trades are short.
- Inverted Head-And-Shoulders: E_1 is a minimum, $E_3 < E_1, E_3 < E_5$, E_1 and E_5 are within 1.5% of their average, E_2 and E_4 are within 1.5% of their average. Pattern gives buy signal, and trades are long.
- Broadening Tops: E_1 is a maximum, $E_1 < E_3 < E_5$, $E_2 > E_4$. Pattern gives sell signal, and trades are short.

- Broadening Bottoms: E_1 is a minimum, $E_1 > E_3 > E_5$, $E_2 < E_4$. Pattern gives buy signal, and trades are long.
- Triangle Tops: E_1 is a maximum, $E_1 > E_3 > E_5$, $E_2 < E_4$. Pattern gives sell signal, and trades are short.
- Triangle Bottoms: E_1 is a minimum, $E_1 < E_3 < E_5$, $E_2 > E_4$. Pattern gives buy signal, and trades are long.
- Rectangle Tops: E_1 is a maximum, tops are within 0.75% of their average, bottoms are within 0.75% of their average, lowest top $>$ highest bottom. Pattern gives buy signal, and trades are long.
- Rectangle Bottoms: E_1 is a minimum, tops are within 0.75% of their average, bottoms are within 0.75% of their average, lowest top $>$ highest bottom. Pattern gives sell signal, and trades are short.
- Double Tops: E_1 is an initial local maximum with index 1 counting from the start of the time subinterval, $E_a = \sup[P_{t_k^*} : t_k^* > t_1^*, k = 2, \ldots, n]$, $t_a^* - t_1^* > 22$, E_1 and E_a are within 1.5% of their average. Pattern gives sell signal, and trades are short.
- Double Bottoms: E_1 is an initial local minimum with index 1 counting from the start of the time subinterval, $E_b = \inf[P_{t_k^*} : t_k^* > t_1^*, k = 2, \ldots, n]$, $t_b^* - t_1^* > 22$, E_1 and E_b are within 1.5% of their average. Pattern gives buy signal, and trades are long.
- Moving-Up-Stream charting type I: E_1 is the minimum, $E_1 < E_3 < E_5$, $E_2 < E_4$, $V_1 > V_3$, $V_2 < V_4$. Pattern gives buy signal, and trades are long.
- Moving-Up-Stream charting type II: E_1 is the minimum, $E_1 < E_3 < E_5$, $E_2 < E_4$, $V_1 >$MA.35, where MA.35 is the average trading volume of the 35 trading minutes within the window. Pattern gives buy signal, and trades are long.

3.3. Procedures and methodology

We fit the general nonparametric regression model 1 using smoothing splines incorporated with GCV criterion to 35 min data (since we collect 1 min data, this consists 35 points), and use the next 30 min data to obtain returns after entering a trade once a pattern has been identified. Therefore, one pattern identification and evaluation procedure requires 65 observations with a total of 43,134 moving windows, i.e. window of point 1 to point 65, window of point 2 to point 66 until window of point 43,134 to point 43,199. We use closing prices as response when fitting the model. Figure 2 gives an example of identification of Triangle Bottom pattern using smoothing splines within a 35 min window on February 6th 2018. The smoothed data provide the local minimum and maximum values $E_1, E_2, \ldots, E_5$ needed to identify the price pattern.

Assuming that we enter trades immediately after identifying a pattern, the closing price is therefore also used as the price to enter a trade. This is a reasonable assumption because of high liquidity and market activity on cryptocurrency exchanges. If a pattern is identified by 35 min data in a particular window, we simulate entering a trade with different holding period, say 1, 2, 3, 4, 5, 10, 15, 20, 25 and 30 min. Our goal is to identify the best holding period for the particular pattern. If a pattern is not identified in a window, we will move to another window fitting. For each pattern identified with different holding period, mean return and sample standard deviation are computed. For example, HS pattern has been identified in 3269 windows (refer to Table 1). Consider a holding period of 20 min, mean return is calculated by the average of these 3269 returns after entering a trade and hold for

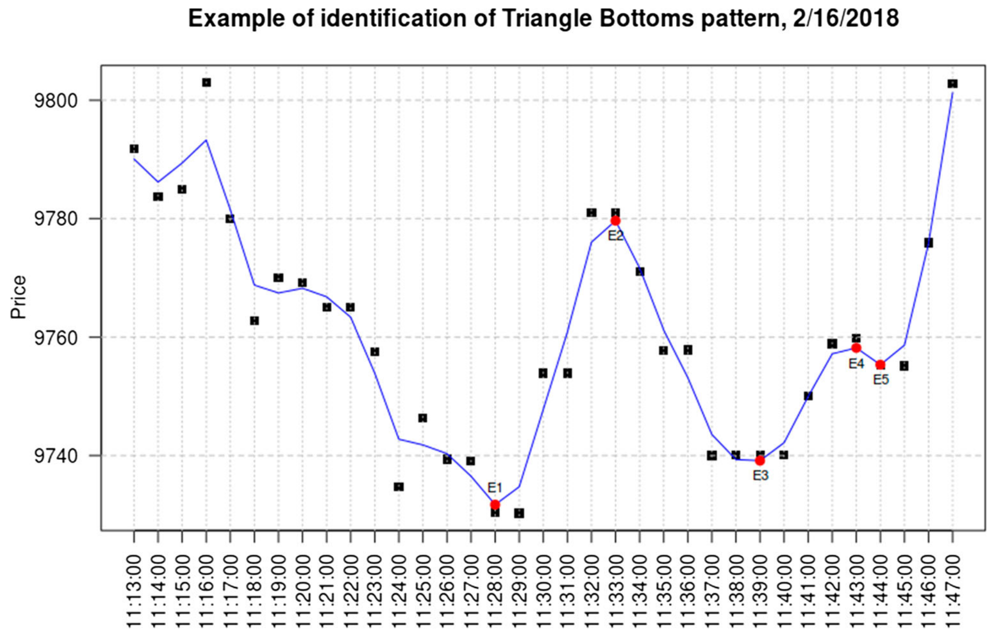

Figure 2. Price pattern: Triangle Bottom.

Table 1. Proportions of returns for baseline (random/unconditional trading strategy) that are above the given technical analysis pattern and holding periods, number of trades and average bitcoin (BTC) volume per minute when trade is taken. The lower the proportion, the higher confidence we have that the strategy has higher returns than the unconditional trading strategy.

	HS	IHS	BT	BB	TT	TB	RT	RB	DT	DB	MUS I	MUS II
1 min	0.0000	0.0000	0.8450	0.8722	0.1845	0.0000	0.9998	1.0000	0.9112	0.6233	0.2414	0.5847
2 min	0.0000	0.0000	0.8474	0.9017	0.3289	0.0000	0.9997	0.9999	0.9859	0.3232	0.2610	0.5071
3 min	0.0000	0.0000	0.7655	0.9399	0.4626	0.0001	0.9999	0.9999	0.7547	0.3475	0.2578	0.2820
4 min	0.0002	0.0000	0.8503	0.8984	0.4083	0.0001	1.0000	1.0000	0.7478	0.2534	0.4778	0.3681
5 min	0.0048	0.0000	0.8527	0.8941	0.4822	0.0000	0.9998	0.9999	0.8446	0.1562	0.7320	0.5186
10 min	0.4920	0.1117	0.5594	0.7252	0.9597	0.0153	0.9626	0.9925	0.8555	0.0390	0.9938	0.8872
15 min	0.9897	0.6407	0.5657	0.9570	0.9817	0.1791	0.7911	0.7681	0.9528	0.1649	0.9927	0.9948
20 min	0.9651	0.9404	0.7719	0.9667	0.9061	0.1371	0.8592	0.5083	0.6803	0.1267	0.9053	0.9964
25 min	0.9336	0.8195	0.7259	0.9498	0.8912	0.2465	0.7403	0.3377	0.7234	0.0524	0.5373	0.9875
30 min	0.7370	0.4468	0.7984	0.9914	0.7711	0.2417	0.9080	0.5674	0.5222	0.0375	0.2692	0.9211
# of trades	3269	3629	218	211	927	1085	5685	6097	13683	14481	1490	528
Mean BTC volume	17.087	16.779	18.663	19.560	19.450	18.265	17.296	15.716	19.266	18.221	20.364	16.383

20 min. Similarly, sample standard deviation is calculated by standard deviation of these 3269 returns.

It is important to evaluate the effectiveness of the proposed methodology. For this purpose, we compare returns due to identified patterns with returns based on random/unconditional trading strategies. Using the number of times the pattern was identified, we draw the same number of random trades from the price data and compute mean return and standard deviation. We perform these draws for 10,000 times to obtain sampling distribution of mean returns due to unconditional trading strategy. This procedure is repeated for each pattern with varies holding periods, as the number of trades is different for each pattern and random trades may yield different results due to different trade lengths. The modified baseline can be interpreted as a strategy of following with the market trend; as trades have no particular pattern, they are able to capture parts of the overall price movement.

For each pattern with certain holding period, we compare the strategy mean return with unconditional mean returns, whose distribution is estimated by sample of 10,000 observations. To this end, we compute proportion of unconditional mean returns that yield better result than the strategy mean return. The lower the proportion, the bigger confidence we have that the particular pattern and holding period form a good basis for a profitable trading strategy. If the strategy does not contribute any information to the analysis of the market, it may be expected that its mean return value will be approximately in the middle of distribution of unconditional returns, such that proportion is close to 0.5. The lower the defined proportion, the higher confidence we have that the strategy has higher returns than the unconditional trading strategy.

4. Analysis results

Table 1 reports mean bitcoin (BTC) trading volume per minute and the proportions of returns for baseline (random/unconditional trading strategy) that are above the given technical analysis pattern at different holding periods. Mean BTC volume is defined as the average volume when trades were entered with different strategies. Consider Head-And-Shoulders (HS) pattern, mean BTC volume is 17.087 BTC per minute, which is calculated by the average of 3269 trading volumes when HS pattern has been identified and trades were entered with certain strategies. Mean BTC volume also gives us advice on how much trading volume we should enter in practice.

Let us now take a look of the proportions reported. For Head-And-Shoulders strategy with holding period of 5 min, proportion of returns of unconditional trading strategy returns that was higher than strategy mean return is 0.0048. We see that strategy effect varies between strategies and for different holding periods. Holding periods of 10 (with proportion of 0.039) and 20 min (with proportion of 0.1267) can be profitable when considering Double Bottoms pattern. The most interesting technical analysis patterns are both Head-And-Shoulders patterns and Triangle Bottoms. These patterns are very effective for short term holding periods. The patterns that deserve attention are rectangular patterns. These patterns have uniformly bad performance for short term holding periods and can become very profitable if we use the opposite interpretation, such that we sell at buy signals and buy at sell signals. This inverse interpretation was previously considered by Thomas [9] for particularly bad-performing patterns. For Double Tops pattern, 2 min holding strategy with opposite interpretation can also be immensely profitable, as there are identified to be 13,683 trades, such that a new trade is entered at approximately every third minute.

After identifying the most interesting strategies: Head-And-Shoulders, Inverted Head-And-Shoulders and Triangle Bottoms, we will evaluate their overall returns. We do not consider compound returns, assuming that we enter each trade with the same amount of capital. It should be noted that bitcoin has increased in price by 30.54% during the period of study. Table 2 gives the mean returns, standard deviations, and combined/cumulative returns of strategies based on the three patterns for holding periods from 1 to 5 min. Note

Table 2. Mean returns, standard deviations, and cumulative/combined returns of strategies based on patterns Head-And-Shoulders, Inverted Head-And-Shoulders and Triangle Bottoms for holding periods from 1 to 5 min.

	HS			IHS			TB		
	Mean	Combined	SD	Mean	Combined	SD	Mean	Combined	SD
1 min	0.0001451	0.4742002	0.0019285	0.0001536	0.5575000	0.0019500	0.0002733	0.2965000	0.0019100
2 min	0.0002148	0.7022002	0.0026354	0.0003059	1.1100001	0.0027028	0.0003236	0.3511000	0.0027792
3 min	0.0002168	0.7087999	0.0032918	0.0003490	1.2665000	0.0033401	0.0003945	0.4280000	0.0033825
4 min	0.0001759	0.5750001	0.0038088	0.0003726	1.3519999	0.0037867	0.0004628	0.5021000	0.0038196
5 min	0.0001455	0.4755999	0.0042598	0.0003436	1.2467999	0.0042912	0.0005088	0.5521000	0.0042520

that combined returns are not compounded. We assume that we always enter trades of the same size. It is a realistic assumption, as there are limitations of trade size due to available orders in order books. However, when compounded, total returns would be far higher than the combined returns.

The results from Table 2 are very promising. With a simple algorithmic strategy, we are able to achieve very high excess returns using well-known technical analysis patterns. For example, Inverted Head-And-Shoulders pattern with holding period of 4 min yields mean return 0.0003726, standard deviation 0.0037867, and combined return of 1.3519999. The total returns of the strategies would be higher if we maximize the long-term wealth from Kelly Criterion [6,14].

5. Conclusion and discussion

In this research, we proposed using smoothing splines to identify technical analysis patterns in bitcoin market. We also proposed a method to evaluate the effectiveness of the technical analysis patterns by market returns. By using one month data from GDAX, we have identified three interesting technical analysis patterns: Head-And-Shoulders, Inverted Head-And-Shoulders and Triangle Bottoms for bitcoin prices. Some patterns, such as Head-And-Shoulder patterns and Triangle Bottoms pattern in U.S. markets have not shown as great returns as we found in bitcoin market. One reason is that stock markets fluctuate rather slowly when compared to cryptocurrencies. Moreover, Head-And-Shoulder pattern is based on short selling and it has yielded very high returns in an otherwise bullish market. It may be the case that both interpretations of Head-And-Shoulder pattern are robust to the overall market trend, but more studies over larger periods of time are needed to make a definite conclusion.

It should be noted that implementation of these strategies may be hindered by market liquidity. Individual returns have high degree of variability; however, we get high returns after a lot of trades are performed. Variability can be considered to be a negligible source of error because the implementation of strategies is relegated to an automated trading system. Our results are promising and could be used as a reference to develop an algorithmic trading strategy.

Future study of algorithmic trading in bitcoin market can be extended by using larger datasets, other technical analysis patterns and different nonparametric regression procedures. Information dataset can be extended by considering finer or rougher grid than 1 min, and incorporating other information such as market news. Analyzing tick data is another possible extension of the study, such that time intervals between trades are not of the same lengths.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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