

Time Series Stat 581, Homework 5
Due on Thursday December 11

1. *This exercise tries to show the importance of detrending data in a periodogram analysis.* For the CO_2 data available in R and WITHOUT applying any type of detrending compute and plot:
 - (a) Raw periodogram ω vs. $I(\omega)$.
 - (b) Bayesian periodogram (in likelihood scale).
 - (c) Autocorrelation function up to lag 100.

Also, look at the residuals of the simple cyclical model discussed in class using $\omega = 2\pi/12$. Do you think the fit of the model is adequate? Are there any obvious cycles noticeable from the different periodograms? Explain and compare the different results.

2. Generalize the cyclical model discussed in class to allow for 2 different frequencies ω_1, ω_2 . Can we still treat this extension as a linear model? If so and assuming $F'F \approx (n/2)I$, find an approximation for $p(x|F)$ that depends on $I(\omega_1)$ and $I(\omega_2)$, the periodogram evaluated at the frequencies ω_1 and ω_2 . No computer is required for this problem.
3. Assume that $X_t = \alpha X_{t-1} + \epsilon_t$, so the time series follows an autoregressive process of order 1. Also assume that $-1 < \alpha < 1$. Using the definitions seen in class, derive an expression for the spectral density of X_t (see class notes). Make plots of this spectral density for values $\alpha = -.8, .25, .9$, plot this density as a function of ω where ω is between 0 and π . Write a brief interpretation of the plots. For this problem, it may help to look at exercise 4.5 in Shumway and Stoffer.
4. Using the 400 observations corresponding to the “short and central” EEG series compute the periodogram and produce a plot of the frequencies versus the values of the periodogram.
 - (a) Plot ω vs. $I(\omega)$.
 - (b) Make a plot of a smoothed and tapered periodogram. You are free to decide on the type/degree of smoothness and tapering.
 - (c) Test the null hypothesis H_o : *observed process is a Normal white noise* using the statistic T based on the maximum periodogram ordinate. Do you reject the null hypothesis?
5. From Shumway and Stoffer’s text Exercise 3.31.