Stat 581: Time Series. Fall 2008 Homework 4.
Due on Thursday November 13, 2008

1. Assume that 100 observations of an AR(2) model $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t$ gave the following sample ACF: $\hat{\rho}_1 = 0.8$, $\hat{\rho}_2 = 0.5$ and $\hat{\rho}_3 = 0.4$. Find estimates for ϕ_1 and ϕ_2.

2. Consider the AR(1) model $(1 - \phi B)(X_t - \mu) = \epsilon_t$ where $\epsilon_t \sim N(0, \sigma^2)$.

   (a) For $\mu = 0$, find the maximum likelihood estimator for ϕ and σ^2. (Find explicit expressions as function of x_t and you may condition on x_1)

   (b) Find the maximum likelihood estimators for ϕ and μ, when $\mu \neq 0$.

3. For the EEG data (400 observations) fit an AR(8) using the Bayesian reference analysis via Monte Carlo simulation as discussed in class (see code5 in the class notes) ONLY draw 2500 samples instead of 5000.

   (a) Show histograms of the simulated coefficients $\phi_1, \ldots, \phi_8$.

   (b) Present histograms for the modulus and wavelength ($\lambda's$) of the complex roots ordered by wavelengths.

   (c) Also compute and show a 95 % posterior probability interval for each of these modulus and wavelengths. (You may use the function *quantile* in R) .

4. Consider the **oxygen-isotope** time series which is part of the data sets in our website.

   (a) Make a time series plot of the data and detect if there is any obvious trend. If so, remove the trend with a lowess estimator.

   (b) Under AR models and using the AIC/BIC criteria, obtain the order p that is the most compatible for this data. Show a plot of AIC/BIC values and identify the optimal value from this plot.

   (c) Fit an AR model using the value of p obtained in (b) via maximum likelihood and obtain the residuals. Plot the ACF, PACF, qqplot and periodogram of your residuals. Are they normally distributed? Do think that this residuals are consistent with white noise process assumption?

   (d) Using your AR/MLE fit find **point estimates** for the modulus and wavelengths for the complex reciprocal roots when these roots are ordered by frequencies (no need to use the Bayesian reference analysis here).

(e) Compute MSE forecasts for the next 100 observations and include 95% prediction intervals. Present a plot with this forecast and intervals.

(f) Conditional on your MLE produce 3 samples from the predictive distribution for the next 100 observations. (For this type of climatogical data the time scale is reversed in time, so when you "forecast" in reality your are predicting the past rather than future.).

5.