STAT 581 Time Series, Homework 3. Fall 2008 Due on Friday October 10. Please turn-in your work at the deparmtment office HUM 4th floor

1. Using the 400 observations corresponding to the "short and central" EEG series compute the periodogram and produce a plot of the frequencies versus the values of the periodogram.

2. For each of the following represenstations, check if the process X_t is stationary and/or invertible. Justify your answer in each case.

   - $(1 - B)X_t = (1 - 1.5B)\omega_t$.
   - $(1 - .8B)X_t = (1 - .5B)\omega_t$.
   - $(1 - 1.1B + .8B^2)X_t = (1 - 1.7B + .72B^2)\omega_t$

3. For an AR(2) process $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t$ with real roots, check that the condition for stationarity $(-1 < \alpha_1 \leq \alpha_2 < 1)$ implies that $\phi_1 + \phi_2 < 1$ and $\phi_2 - \phi_1 < 1$. Also show that the partial autocorrelation for this process has the form

$$P_1 = \phi_1/(1 - \phi_2); \quad P_2 = \phi_2; \quad P_k = 0 \; with \; k \; \geq \; 3$$

4. Suppose X_t follows an MA process of order 2. That is $X_t = \omega_t - \theta_1 \omega_{t-1} - \theta_2 \omega_{t-2}$ Find the theoretical autocorrelation function of the process.

5. Generate 500 observations from a stationary AR(4) process with two complex pairs of conjugate reciprocal roots that have the following conditions. The first pair has modulus $r_1 = .9$ and frequency $\omega_1 = .5$; the second pair has modulus $r_2 = 0.75$ and frequency $\omega_2 = 1.35$. Plot the simulated process and also the acorresponding ACF and PACF.

6. Consider the infinite order MA process defined by

$$X_t = \omega_t + C(\omega_{t-1} + \omega_{t-2} + \ldots)$$

   where C is a constant and the $\omega_t's$ are iid $N(0, \sigma^2)$ random variables.

   (a) Show that the process X_t is non-stationary.

(b) Consider the series of first differences $Y_t = X_t - X_{t-1}$. Show that Y_t is a first order MA process (MA(1)). What is the MA (θ) parameter of this process?

(c) Find the range of values for C for which this MA process is invertible .

(d) Find the autocorrelation function of Y_t.

7. Suppose that we generated 400 observations of a time series process. The first 3 sample autocorrelations ($r(k)$) are $r(1) = 0.06$, $r(2) = 0.18$, $r(3) = -0.22$. Do we have evidence that this simulated data does not follow a white noise process?

8. For the following statements answer TRUE or FALSE and justify your answer completely. Provide examples or counterexamples as you see fit.

   (a) A time series process with a constant variance is always a second order stationary process.

   (b) A time series data looks periodic and with an increasing mean level. The model $X_t = a + bt + \epsilon_t$ always gives an adequate representation to the trend of the data.

   (c) Before applying any time series method, we should always take a first difference of the data. This assures stationarity of the process and then we can apply any analysis we desire, like estimating a periodogram or fitting an AR model.