STA 590, Statistical Computing, Spring 2005

HW5 Due date: April 7, 2005

1. Consider that the target distribution $\pi(\theta)$ is $N(0,1)$ pdf, then

$$\pi(\theta) = \frac{1}{\sqrt{2\pi}} exp(-\frac{\theta^2}{2})$$

   (a) Define completely (i.e. determine transition kernel and acceptance probability) of the Metropolis-Hastings algorithm assuming that the proposal distribution $q(\theta, \theta')$ is $U(-\delta, \delta)$, where δ is a pre-specified quantity.

   (b) Repeat the previous item but using a proposal distribution that is $U(\theta - \delta, \theta + \delta)$.

   (c) Which of the two proposals distributions will be better in practice an why? (Justify your answer without using the results from the next item)

   (d) Write some code that implements the Metropolis-Hastings algorithm with both proposals. Use $\delta = 0.5$ and set your initial value to -1. In each case, run the algorithm 5000 times and compute the acceptance rate. Did it converge to the target distribution? Compare the results from the two proposals. Use trace plots, histograms or some other graphical device that you consider appropriate to justify your answers. *P.S. If you wish to modify the value of δ to improve acceptance rates, this will be fine.*

```
# Here is a sketch for the M-H algorithm  N(0,1) distribution is target
delta <- 0.5
accept <- 0
M <- 5000
th <- -1
xs <- th
for(i in 1:M){
  u <- runif(1)
# Generate value thnew from proposal
# compute acceptance probablity
  if (u < acceptance){
      th <- thnew
      accept <- accept + 1
     }
   xs <-c(xs,th)
}
```

2. Consider the model

$$y_i = Ae^{\lambda t_i} + \epsilon_i, i = 1, 2, \ldots, n$$

with $\epsilon_i \sim N(0, \sigma^2)$ iid random variables where σ^2 is known and t_i are known times of observation. Assume that A and λ are apriori independent $A \sim N(\mu, \sigma_0^2)$; $\lambda \sim Ga(\alpha, \beta)$ where all the quantities μ, σ_0^2, α and β are specified.

(a) Formulate a Metropolis-Hastings algorithm with a Random-Walk proposal to produce draws for the posterior distribution of A and λ.

(b) Simulate $n = 200$ observations of y_i from the model with $A = 0.5$, $\lambda = 1$, $t_i = i$ and $\sigma^2 = 0.5$. Test your M-H algorithm from (a) assuming that the prior hyperparameters are $\mu = 0$, $\sigma_0^2 = 2$, $\alpha = \beta = 1$