Statistical Inference 453/553. Spring 2003

Homework 4

April, 2003

1. **Exercise 7.47** Let r be the radius of the circle and X_i; $i = 1, \dots n$ the i measurement of this radius. Then $X_i \sim N(r, \sigma^2)$. We need an unbiased estimator for $A = \pi r^2$. Remember that $E(\bar{X}) = r; E(S^2) = \sigma^2$. Furthermore, $E(\bar{X}^2) = r^2 + \sigma^2/n$. An unbiased estimator for A is

$$\hat{A} = \pi(\bar{X}^2 - S^2/n)$$

.

Notice that $\hat{A}$ is a function of the sufficient and complete statistic $T = (\bar{X}, S^2)$. By the Lehmann-Scheffé Theorem, $\hat{A}$ is best unbiased.

(Observation: If σ^2 is assumed known, then $\hat{A} = \pi(\bar{X}^2 - \sigma^2/n)$. Although, there is no reason to assume that σ^2 is known.)

2. **Exercise 7.58** Part (a). We have that $logf(x|\theta) = |x|log(\theta/2) + (1 - |x|)log(1 - \theta)$. Then,

$$\frac{dlogf(x|\theta)}{d\theta} = |x|(1/\theta) - (1 - |x|)(1/(1 - \theta) = 0$$

which implies that the MLE is $\hat{\theta} = |X|$.

Part(b) $E(T(X)) = 2P[X = 1] = 2(\theta/2) = \theta$. $T(X)$ is unbiased.

Part(c) In a previous exercise, we showed that $|X|$ is sufficient and complete. Notice that

$$E(|X|) = 2(\theta/2) + 0(1 - \theta) = \theta$$

Then $T = |X|$ is an unbiased estimator of θ. By the Lehmann-Scheffé theorem, T is an UMVUE so it is best than any other unbiased estimator.