Department of Mathematics and Statistics
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Homework 1 **Math 510: Real Analysis 1** **Fall 2009**

Instructions: Complete all six problems to get full credit. The exercises are in Chapter 1 of our textbook Rudin's *Principles of Mathematical Analysis*. This homework is due on Tuesday September 1st, 2009.

1. (#3 in Rudin) Prove Proposition 1.15: The axioms of multiplication imply the following statements for all x, y, and z in a field F.

(a) If $x \neq 0$ and $xy = xz$ then $y = z$.

(b) If $x \neq 0$ and $xy = x$ then $y = 1$.

(c) If $x \neq 0$ and $xy = 1$ then $y = 1/x$.

(d) If $x \neq 0$ then $1/(1/x) = x$.

Note that (b) and (c) assert the uniqueness of the multiplicative identity and the multiplicative inverse respectively.

2. (#5 in Rudin) Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$, where $x \in A$. Prove that

$$\inf A = -\sup(-A).$$

Where the $\inf A$ is defined to be the greatest lower bound of A.

3. (#8 in Rudin) Prove that no order can be defined in the complex field that turns it into an ordered field. *Hint:* -1 is a square.

4. (#9 in Rudin) Suppose $z = a + ib$, $w = u + iv$, and

$$a = \left(\frac{|w| + u}{2}\right)^{1/2}, \qquad b = \left(\frac{|w| - u}{2}\right)^{1/2}.$$

Prove that $z^2 = w$ if $v \geq 0$ and that $(\overline{z})^2 = w$ if $v \leq 0$. Conclude that every complex number (with one exception!) has two complex square roots.

5. (#13 in Rudin) If x, y are complex, prove that

$$\big|\, |x| - |y| \,\big| \leq |x - y|.$$

6. (#15 in Rudin) Under what conditions does equality hold in the Cauchy-Schwarz inequality? Recall that the aformentioned inequality says that if $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ are complex

numbers, then

$$\left|\sum_{j=1}^{n} a_j \overline{b_j}\right|^2 \leq \sum_{j=1}^{n} |a_j|^2 \sum_{j=1}^{n} |b_j|^2.$$