Every interval in $\mathbb{R}$ of the form $I = [a, b)$ belongs to infinitely many random dyadic grids $\mathcal{D}^{r,\beta}$. The scaling parameter is unique, and the random parameter $\beta = \{\beta_i\}_{i\in\mathbb{Z}}$ is determined for all integers i less than $-m$, where m is a fixed integer to be determined below. For $i \geq -m$, β_i can be 0 or 1, since there is a binary choice for the parent of I: the parent could be the interval $[a, b + |I|)$ so that I is the right child, or it could be $[a - |I|, b)$ so that I is the left child, likewise I will have four choices for grandparent (each parent has two choices for their respective parent), etc.

To be more precise, let $|I| = b - a > 0$ and let m be the unique integer such that $2^{-m} \leq |I| < 2^{-m+1}$. Let $r := 2^m|I| = 2^m(b - a)$, by our choice of m, $1 \leq r < 2$ and this will be the scaling parameter for all grids that contain the interval I. There is also a unique integer k such that $k2^{-m} \leq a/r < (k + 1)2^{-m}$. Let $x_m = a/r - k2^{-m}$, by our choice of k then $0 \leq x_m < 2^{-m}$. Moreover $a = rx_m + rk2^{-m}$ and $|I| = r2^{-m}$ therefore

$$I = a + [0, |I|) = r(x_m + k2^{-m} + [0, 2^{-m})) = r(x_m + [k2^{-m}, (k + 1)2^{-m}).$$

So that $I = r(x_m + J)$ where $J := [k2^{-m}, (k + 1)2^{-m}) \in \mathcal{D}^0_m$. The non-negative real number x_m has a binary expansion of the form $x_m = \sum_{i<-m} \beta'_i x^i$, hence $I \in \mathcal{D}^{r,\beta}_m$ for all $\beta = \{\beta_i\}_{i\in\mathbb{Z}} \in \{0, 1\}^{\mathbb{Z}}$ such that $\beta_i = \beta'_i$ for all $i \leq -m$.

When $r = 1$ and $\beta = \{0\}_{i\in\mathbb{Z}}$, the initial grid is the standard grid $\mathcal{D}^{1,0} = \mathcal{D}^0$. Then $\mathcal{D}^{\tau_h(1,0)} = \mathcal{D}^0 + h$, the standard grid translated by h; $\mathcal{D}^{\delta_a(1,0)} = (1/a)\mathcal{D}^0$, the standard grid dilated by the reciprocal of a; and $\mathcal{D}^{(1,0)^\sim} = \breve{\mathcal{D}}^0$ is the mirror standard grid.