Very nice!

PROFESSOR PEREYRA,

When given the proof exercise in class (MATH 401) yesterday, I realized I proved the converse of "Jonathan's Lemma" instead of the actual lemma itself. ✓ ü

I know this submission is not for a grade but here is my submission of the following lemma's (including "Jonathan's Lemma")

Thanks I really appreciate this. Keep doing in the future if you realize something you have submitted could be done better.

1) Cristina's Lemma: If p is even, then p^2 is even.

2) Jonathan's Lemma: If p^2 is even, then p is even.

3) Jordan's Lemma: If p is odd, then p^2 is odd.

4) Someone's Lemma: If p^2 is odd, then p is odd ü

We did this in recitation! This is great.

This submission is merely for the correction for yesterday's mistake and ultimately for proof writing practice.

See you in class ü

Jonathan
Ortiz

Very nice

Thanks I
really appreciate
this, keep doing
in the right things
because everything
you have everything
will be fine
be you.

Jonathan Ortiz
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Cristina's Lemma:

THM: If p is even, then p^2 is even.

In $p \to q$ format where $p := p$ is even and $q := p^2$ is even. Side note: the symbol ":=" means "is defined to be" or "by def'n".

Proof: Let p be an integer ($p \in \mathbb{Z}$) and p be even, i.e. p can be written as: $p := 2k$ where $K \in \mathbb{Z}$.

By the above definition $p := 2k$ so therefore $p^2 = (2k)^2$. By arithmetic, $p^2 = 4k^2$ and by the properties of associative and commutativity $4k^2$ can be rearranged and written as $2(2k^2)$. Denote $p^2 = 2(2K^2)$ as equation (1).

We now have that $p^2 = 2(2k^2)$. Let's examine $2K^2$. By properties of integers, an integer multiplied by another integer is still an integer. Therefore $K \cdot K = K^2$ is an integer. Furthermore, $2 \in \mathbb{Z}$, so $2 \cdot K^2 = 2K^2$ is an integer, i.e. $2K^2 \in \mathbb{Z}$

$\therefore$ Let's define $2K^2$ to be A where $A \in \mathbb{Z}$

By making a substitution in equation (1), we can write $p^2 = 2(A)$ and by observation, 2 divides p^2 ($2 | p^2$). perfect

We can therefore conclude that p^2 is an even number.

We have established that p is even (by definition) and have discovered (through arithmetic, substitution and proof techniques) that p^2 is even.

We can now conclude that our Lemma (Cristina's Lemma) that p being even does in fact imply that p^2 is even.

$\therefore$ If p is even, then p^2 is even is true and was proven above. $\blacksquare$

Jonathan's Lemma:

If p^2 is even, then p is even.

This statement is in the $p \to q$ format where $p :=$ p^2 is even and $q :=$ p is even i.e. $p :=$ antecedent and $q :=$ the consequent. Side note: antecedent is synonymous with hypothesis and consequent is synonymous with conclusion.

Proof: We will start by proving (by truth tables) that $p \to q$ is logically equivalent to $\neg q \to \neg p$ i.e. $p \to q \iff \neg q \to \neg p$

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$\to p \to q \iff \neg q \to \neg p$

perfect!

$\therefore$ We can negate p and q and prove the contrapositive i.e. proving $\neg q \to \neg p$ suffices in place of proving the direct implication $p \to q$

Returning to Jonathan's Lemma

$p :=$ p^2 is even { $\neg p :=$ p^2 is odd
$q :=$ p is even { $\neg q :=$ p is odd

As in words, $\neg q \to \neg p$ translates to:

If p is odd, then p^2 is odd

Let's prove this. Let p be an integer ($p \in \mathbb{Z}$) and let p be odd i.e. p can be written in the form
$p := 2K+1$ where $K \in \mathbb{Z}$

If $p := 2K+1$ then $p^2 \equiv (2K+1)^2$. By arithmetic, $p^2 = 4k^2 + 4k + 1$

By regrouping and algebraic rules,

$p^2 = 4K^2 + 4k + 1$
$= 2(2K^2+2K) + 1$ denote this equation (2)

We established in the proof of Cristina's Lemma that two multiplied integers will generate an integer answer. Integers are also closed under addition, i.e. $s, t \in \mathbb{Z}$ $s + t = w$ and $w \in \mathbb{Z}$.

Returning to equation (2), let's examine

$2(2K^2+2K)+1$

$2K^2 \in \mathbb{Z}$ (established in Cristina's Lemma) and $2K \in \mathbb{Z}$ (also established in Cristina's Lemma).

Since the integers are closed under addition, $\therefore (2K^2 + 2K) \in \mathbb{Z}$. Let's denote $(2K^2 + 2K)$ as B $(B := (2K^2+2K))$ where $B \in \mathbb{Z}$

We can now make a substitution into equation (2)

$p^2 = 2(2K^2+2K)+1 = 2(B)+1$

And $2B+1$ is clearly an odd integer.

By definition p is odd ($p := 2K+1$, $K \in \mathbb{Z}$)

Through arithmetic, substitution, and alternate proof techniques, we found that p^2 is odd.

$\therefore$ We have shown that p being odd does imply that p^2 is odd and are forced conclude that the implication, "If p is odd, then p^2 is odd" is true and proven above. We also proved that "If p is odd, then p^2 is odd" $(\neg q \to \neg p)$ is logically equivalent to "If p^2 is even, then p is even" $(p \to q)$ i.e. $p \to q \Leftrightarrow \neg q \to \neg p$. We are also forced to accept the implication, "If p^2 is even, then p is even" as true and proven above as well. With that said "If p^2 is even, then p is even" and "if p is odd, then p^2 is odd" are true statements. $\blacksquare$

Jordans Lemma:

"If p is odd, then p^2 is odd"

In Jonathans Lemma, we proved that the implication, "If p^2 is even, then p is even" is logically equivalent to Jordans Lemma.

Since Jordans Lemma $\Leftrightarrow$ Jonathan's Lemma and we proved Jonathan's Lemma, that suffices to show that Jordan's Lemma is true and proved.

Yes

Someone's Lemma:

If p^2 is odd, then p is odd

This implication is in the $p \to q$ form where $p := p^2$ is odd and $q := p$ is odd.

We proved that the contrapositive is logically equivalent to the direct implication i.e. $p \to q \Leftrightarrow \neg q \to \neg p$.

Let's find the contrapositive of Someone's Lemma,

$p := p^2$ is odd $\;\;\neg p := p^2$ is even
$q := p$ is odd $\;\;\neg q := p$ is even

$\therefore$ The contrapositive, in words, reads:

If p is even, then p^2 is even.

Notice this implication is Cristina's Lemma and we have already proven Cristina's Lemma.

$\therefore$ Cristina's Lemma $\Leftrightarrow$ Someone's Lemma

and we must accept that Someone's Lemma

is true and proved.

All in all we, have established and proven 4 things:

1) If p is even then p^2 is even
2) If p^2 is even then p is even
3) If p is odd then p^2 is odd
4) If p^2 is odd then p is odd.

For 1) and 2), Notice how they are converses of one another i.e. $p \to q$ and $q \to p$

And also examine 3) and 4) which are also converses of one another i.e. $p \to q$ and $q \to p$.

The double implication for 1) and 2) means that we can correctly write (since we have already proven it):

"P is even if and only if p^2 is even"

and 3) and 4) allows us to write

"p is odd if and only if p^2 is odd"

∴ with the above proven implications we have established two two-way implications (properly known as definitions in mathematics).

∴ Once again, to reiterate, "p is even iff p^2 is even" and "p is odd iff p^2 is odd" are both true definition with sound derived proof above.