Exam # 2 - MATH 401/501 - Fall 2016
November 9, 2016

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There are 2 logic problems, 6 shorter problems (true or false,) and 2 longer ones for a total of 80 points. Read carefully all the problems. Please number your problems, name all the pages and enumerate them. You can use your book but not your notes. Do as much as you can in the 50 minutes you have and turn in your work at the end of the exam. Later today, I will email you the exam, please turn in your revised problems on Friday November 11th, 2016 no later than 5pm. GOOD LUCK!

1. (5 points) Write the negation of the following statement:

   For all $\epsilon > 0$ *there is* $N > 0$ *such that for all* $n \geq N$ *we have* $|a_n| \leq \epsilon$.

   The given statement is the definition of:

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2. (5 points) Write the converse of the following statement:

   If a sequence of real numbers is convergent then it is a bounded sequence.

   Is the converse a true statement? TRUE ☐ FALSE ☐

3. (30 points) Decide whether the following statements are true or not. Give a short explanation for your answer (for example: a quick proof or a counterexample).

   If a statement is false, what can you change/add to make it into a true statement?

   (a) Let A and B be nonempty and bounded subsets of real numbers.
   If $A \subset B$ then $\sup A \leq \sup B$.

   TRUE ☐ FALSE ☐

   (b) Given a bounded sequence of real numbers $\{x_n\}_{n\geq 0}$ then $L = \limsup x_n$ is an upper bound for the sequence.

   TRUE ☐ FALSE ☐

(c) Let $\{a_n\}_{n\geq 0}$ be a sequence of real numbers, then $\lim_{n\to\infty} a_n = 0$ if and only if $\lim_{n\to\infty} |a_n| = 0$.

TRUE ☐ FALSE ☐

(d) Given two convergent sequences of real numbers $\{a_n\}_{n\geq 0}$ and $\{b_n\}_{n\geq 0}$ then the sequence $\{2a_n + 3b_n\}_{n\geq 0}$ is also convergent.

TRUE ☐ FALSE ☐

(e) If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

TRUE ☐ FALSE ☐

(f) The series $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{3^n}$ converges.

TRUE ☐ FALSE ☐

4. (20 points) Given two Cauchy sequences of real numbers, $\{a_n\}_{n\geq 0}$ and $\{b_n\}_{n\geq 0}$, show that the sequence of their products, $\{a_n b_n\}_{n\geq 0}$, is convergent.

5. (20 points) A sequence of real numbers is defined recursively by $x_0 := 1$ and if $x_n \in \mathbb{R}$ then $x_{n+1} := \frac{x_n+1}{4}$ for $n \geq 0$.

   (a) Show that the sequence is decreasing, that is $x_{n+1} \leq x_n$ for all $n \in N$.

   (b) Show that the sequence is bounded below.

   (c) Show that the sequence is convergent. Find the limit.

   (d) Find $\inf\{x_n\}_{n\geq 0}$.