MATH 311 — ~~Homework Week 1 Aug 22~~ — SOLUTIONS

Section 1.9 HOMEWORK #3

$\boxed{\text{p 34 \# 13}}$ Determine s and t so that $\vec{C} - s\vec{A} - t\vec{B}$ is perpendicular to both A & B given that

#3

$$\vec{A} = \vec{i} + \vec{j} + 2\vec{k} = (1, 1, 2)$$
$$\vec{B} = 2\vec{i} - \vec{j} + \vec{k} = (2, -1, 1)$$
$$\vec{C} = 2\vec{i} - \vec{j} + 4\vec{k} = (2, -1, 4)$$

$$\vec{V} = \vec{C} - s\vec{A} - t\vec{B} = (2,-1,4) - s(1,1,2) - t(2,-1,1)$$
$$\vec{V} = (2-s-2t, -1-s+t, 4-2s-t)$$

If $\vec{V} \perp \vec{A}$ and $\vec{V} \perp \vec{B}$ then

$\vec{V} \cdot \vec{A} = 0$ and $\vec{V} \cdot \vec{B} = 0$ but

$$\vec{V} \cdot \vec{A} = (2-s-2t) + (-1-s+t) + 2(4-2s-t)$$
$$= (2-1+8) + s(-1-1-4) + t(-2+1-2)$$
$$= 9 - 6s - 3t = 0$$

$$\vec{V} \cdot \vec{B} = 2(2-s-2t) - (-1-s+t) + (4-2s-t)$$
$$= (4+1+4) + s(-2+1-2) + t(-4-1-1)$$
$$= 9 - 3s - 6t = 0$$

$\therefore$ s, t are solutions ~~to~~ the system

$$\begin{cases} 6s + 3t = 9 \\ 3s + 6t = 9 \end{cases}$$

You can verify that $\boxed{s = t = 1}$ is the unique solution.

$\boxed{\text{p39 \#3}}$ Eqn of plane perpendicular to $\vec{D} = (10, -10, 5) = \vec{n}$ Through $P = (1, 1, -3)$ $\quad \vec{V} = (x, y, z)$

#3

$\pi : (\vec{V} - P) \cdot \vec{D} = 0 \qquad \vec{V} - P = (x-1, y-1, z+3)$

$$\boxed{10(x-1) - 10(y-1) + 5(z+3) = 0}$$

$$\boxed{10x - 10y + 5z = -15} \longrightarrow \boxed{2x - 2y + z = -3}$$

Most simplified ANS.

MATH 311 — ~~Homework Work~~ — SLNS | HOMEWORK #3 |

Section 1.10 HOMEWORK #3

| p. 39 #11 |

Angle that the plane OAB makes with z-axis, if $A = (1,3,2)$ $B = (2,1,1)$

#3

We want the angle θ between the normal to the plane OAB and $\vec{k} = (0,0,1)$. $\boxed{\text{ANS: } 90-\theta^\circ}$ vector $\vec{n}$

In parametric form the plane OAB

$$\Pi \qquad \vec{p} = \vec{o} + t\vec{A} + s\vec{B}$$

Note: If you remember cross pro can use $\vec{n} = \vec{A} \times \vec{B}$!

$$\begin{cases} ① \ x = t + 2s \\ ② \ y = 3t + s \\ ③ \ z = 2t + s \end{cases}$$

We are going to reduce this 3 equs in 5 variables to 1 equ in only the variables x, y, z: The eqn of the plane OAB. Then can read from coeffs. the normal vector $\vec{n}$

From ① $t = x - 2s$

plug into ② $y = 3x - 6s + s$

$5s = 3x - y$

$s = \dfrac{3x-y}{5}$

plug into $t = x - 2\dfrac{(3x-y)}{5} = \dfrac{5x-6x+2y}{5}$

plug into ③ $t = \dfrac{2y-x}{5}$

$$z = 2\left(\frac{2y-x}{5}\right) + \left(\frac{3x-y}{5}\right) = \frac{4y-2x+3x-y}{5}$$

$$5z = 3y + x \implies \boxed{x + 3y - 5z = 0}$$

non-parametric eqn of the plane OAB

$$\vec{n} = (1, 3, -5)$$

$$\cos\theta = \frac{\vec{n}\cdot\vec{k}}{|\vec{n}|\,|\vec{k}|} = \frac{-5}{\sqrt{1^2+3^2+(-5)^2}} = \frac{-5}{\sqrt{35}} = -\sqrt{\frac{5}{7}}$$

$|\vec{k}| = 1$

$$\boxed{\theta = \arccos\left(-\sqrt{\frac{5}{7}}\right)} = \cos^{-1}\left(-\sqrt{\frac{5}{7}}\right)$$

$\boxed{\text{ANS: } 90^\circ - \cos^{-1}\left(\sqrt{\frac{5}{7}}\right)}$