

Section 2.4

p.102 #2

Find \vec{v} , \vec{a} if

$$r = b(1 + \sin t) \quad \theta = e^{-t} - 1$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt} \vec{u}_r + r \frac{d\theta}{dt} \vec{u}_\theta$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left[\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right] \vec{u}_r + \left[r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right] \vec{u}_\theta$$

In our case

$$\frac{dr}{dt} = b \cos t$$

$$\frac{d\theta}{dt} = -e^{-t}$$

$$\frac{d^2 r}{dt^2} = -b \sin t$$

$$\frac{d^2 \theta}{dt^2} = e^{-t}$$

so

$$\vec{v} = b \cos t \vec{u}_r + r(-e^{-t}) \vec{u}_\theta = \boxed{b \cos t \vec{u}_r - r e^{-t} \vec{u}_\theta = \vec{v}}$$

$$\vec{a} = \boxed{-b \sin t - r(-e^{-t})^2 \vec{u}_r + [r e^{-t} + 2 b \cos t (-e^{-t})] \vec{u}_\theta}$$

$$\boxed{\vec{a} = [-b \sin t - r e^{-2t}] \vec{u}_r + [r e^{-t} - 2 b \cos t e^{-t}] \vec{u}_\theta}$$

p102 #8

A disc rotates back and forth with angular

velocity $\frac{d\theta}{dt} = \cos t \text{ rad/sec}$. An insect starting at

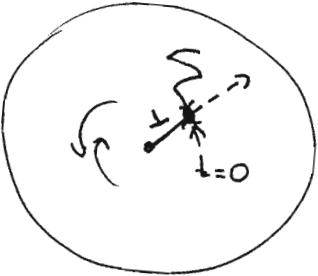
1 cm from center of disc at time $t=0$ crawls

outward at a rate of 2t cm/sec,

so $\frac{dr}{dt} = 2t \text{ cm/sec}$. Find the position,

velocity ~~\vec{v}~~ , and speed of the insect

after 3 sec. $R(t)$, $V(t)$, $|V(t)|$



P.102 #8 (cont)

Position

$$\vec{R} = r \vec{u}_r$$

Since $\frac{dr}{dt} = 2t \Rightarrow r = t^2 + c$

since at $t=0$ $r=1$

$$\Rightarrow c=1$$

So $r(t) = t^2 + 1 \quad \therefore \vec{R}(t) = (t^2 + 1) \vec{u}_{r(t)}$

$$\begin{aligned} \vec{R}(2) &= (2^2 + 1) \vec{u}_{r(2)} = 5 \vec{u}_{r(2)} \\ &= 5 (\cos \theta(2) \vec{i} + \sin \theta(2) \vec{j}) \end{aligned}$$

Now $\frac{d\theta}{dt} = \cos t \Rightarrow \theta = \sin t + c$

can assume that at $t=0$ $\theta(0)=0 \Rightarrow c=0$

$\theta(2) = \sin 2 \Rightarrow R(2) = 5 [\cos(\sin 2) \vec{i} + \sin(\cos 2) \vec{j}]$

$$\vec{v}(2) = \frac{dr(2)}{dt} \vec{u}_r + 2 \frac{d\theta(2)}{dt} \vec{u}_\theta, \text{ speed}$$

$$\boxed{\vec{v}(2) = 4 \vec{u}_r + 2 \cos 2 \vec{u}_\theta}$$

$$\begin{aligned} |\vec{v}(2)| &= \sqrt{4^2 + 4 \cos^2 2} = \\ |\vec{v}(2)| &= 2 \sqrt{1 + \cos^2 2} \end{aligned}$$

P 102 # 14 Find $\frac{d^3 \vec{R}}{dt^3}$ in terms of \vec{u}_r and \vec{u}_θ

$$\frac{d^3 \vec{R}}{dt^3} = \frac{d}{dt} \left[\frac{d^2 \vec{R}}{dt^2} \right] = \frac{d}{dt} \left[\left(\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right) \vec{u}_r + \left(r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \vec{u}_\theta \right]$$

$$= \left(\frac{d^3 r}{dt^3} - \frac{dr}{dt} \left(\frac{d\theta}{dt} \right)^2 - r \frac{d}{dt} \left(\frac{d\theta}{dt} \right)^2 \right) \vec{u}_r + \left(\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right) \frac{d}{dt} \vec{u}_r$$

$$+ \left(\frac{dr}{dt} \frac{d^2 \theta}{dt^2} + r \frac{d^3 \theta}{dt^3} + 2 \frac{d^2 r}{dt^2} \frac{d\theta}{dt} + 2 \frac{dr}{dt} \frac{d^2 \theta}{dt^2} \right) \vec{u}_\theta + \left(r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \vec{u}$$

Remember that $\frac{d\vec{U}_r}{dt} = \frac{du_r}{d\theta} \cdot \frac{d\theta}{dt} = \frac{d\theta}{dt} u_\theta$
 $\frac{d\vec{U}_\theta}{dt} = \frac{du_\theta}{d\theta} \cdot \frac{d\theta}{dt} = -\frac{d\theta}{dt} u_r$

hence $\frac{d^3 R}{dt^3} = \left(\frac{d^3 r}{dt^2} - \frac{dr}{dt} \left(\frac{d\theta}{dt} \right)^2 - r^2 \frac{d^2 \theta}{dt^2} \frac{d\theta}{dt} \right) \vec{U}_r$
 $+ \left(\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right) \frac{d\theta}{dt} \vec{U}_\theta$
 $+ \left(\frac{dr}{dt} \frac{d^2 \theta}{dt^2} + r \frac{d^3 \theta}{dt^3} + \frac{2dr}{dt} \frac{d\theta}{dt} + \frac{2r}{dt} \frac{d^2 \theta}{dt^2} \right) \vec{U}_\theta$
 $+ \left(r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \left(-\frac{d\theta}{dt} \right) \vec{U}_r$

$$\boxed{\frac{d^3 R}{dt^3} = \left(\frac{d^3 r}{dt^2} - 3r \frac{d^2 \theta}{dt^2} \frac{d\theta}{dt} - 3 \frac{dr}{dt} \left(\frac{d\theta}{dt} \right)^2 \right) \vec{U}_r$$

 $+ \left(3 \frac{d^2 r}{dt^2} \frac{d\theta}{dt} - r \left(\frac{d\theta}{dt} \right)^2 + 3 \frac{dr}{dt} \frac{d^2 \theta}{dt^2} + r \frac{d^3 \theta}{dt^3} \right) \vec{U}_\theta}$