Compute \( \nabla f = \nabla f \) for \( f = \sin x + e^{xy} \).

\[
\begin{align*}
\frac{\partial f}{\partial x} &= \cos x + ye^{xy} \\
\frac{\partial f}{\partial y} &= xe^{xy} \\
\frac{\partial f}{\partial z} &= 1 \\
\end{align*}
\]

\[\nabla f = (\cos x + ye^{xy}, xe^{xy}, 1)\]

\# 9
\[f(x, y, z) = x + xy + z\]

Find \( D_u f \) at point \((1, -2, 2)\).

(a) For \( u \parallel 2i + 2j - k \Rightarrow u = \sqrt{9} = 3\)

(b) For \( u \parallel 2i + 2j + k \Rightarrow u = \sqrt{9} = 3\)

(c) \( u = \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right) \) and \( D_u f = \nabla f (1, -2, 2) \cdot u \)

\[\nabla f = (1 + y^2, x^2, 1)\]

\[\nabla f (1, -2, 2) = (-3, 2, -2)\]

\[D_u f = (-3, 2, -2) \cdot \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right) = \frac{-6 + 4 - 2}{3} = \frac{-4}{3} = D_u f\]

(5) \( u = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) \)

\[D_u f = \nabla f (1, -2, 2) \cdot u = (-3, 2, -2) \cdot \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) = \frac{-6 + 4 - 2}{3} = \frac{-4}{3} = D_u f\]

### #12

In next page:

### #14

Consider \( f(x, y, z) = x^2 + y^2 + z \), then \( f(2, 1, 1) = 5\)

\[\nabla f (2, 1, 1) \perp \text{level surface } f(x, y, z) = f(2, 1, 1)\]

That is \( x^2 + y^2 = 5 \)

\[\nabla f = (2x, 2y, 1)\]

\[\hat{n} = \nabla f (2, 1, 1) = (4, 1, 1) = \hat{n}\]

Any scalar multiple of \((4, 1, 1)\) is a normal to the surface.
**Section 3.1**

**#12**

Direction of maximal increase of $f(x, y, z) = e^{-xy} \cos z$ at $(1, 1, 0)$

**ANS**

$$\nabla f(1, 1, 0)$$

$$\nabla f = (-y e^{xy}, -x e^{xy} \cos z, -e^{-xy} \sin z)$$

$$\nabla f(1, 1, 0) = (-e^{-2}, -e^{-2}, 0)$$

**#27**

At what angle does the curve $x = t, y = 2t - t^2, z = 2t^2$ intersect the surface $x^2 + y^3 + 3z^2 = 14$ at point $(1, 1, 2)$?

**Tangent plane to the surface with normal $n$.**

We want the angle $90^\circ - \theta$ where $\theta$ is the angle between $n$ and the tangent to the curve at the point $(1, 1, 2) = R(1) = (1, 0, 8)$.

As in exercise 14, consider $f(x, y, z) = x^2 + y^3 + 3z^2$.

Then $f(1, 1, 2) = 1 + 1 + 12 = 14$.

So the surface is the level surface of $f$.

Then $\nabla f(1, 1, 2) = (2, 3, 12) = n$.

$$\nabla f = (2x, 3y, 6z)$$

$$\nabla f(1, 1, 2) = (2, 3, 12)$$

Tangent to the curve is $R'(t) = (1, 2 - 2t, 8t)$.

$R(1) = (1, 1, 2)$,

$R'(1) = (1, 0, 8)$

$|R'(1)| = \sqrt{1 + 0 + 64} = 8$

$\cos \theta = \frac{\nabla f \cdot R'(1)}{|\nabla f| \cdot |R'(1)|} = \frac{2 + 0 + 96}{\sqrt{2} \cdot 8} = \frac{98}{8\sqrt{2}} = \frac{98}{16\sqrt{2}} = \frac{98}{28\sqrt{2}} = \frac{98}{16\sqrt{2}}$

**ANS**

$90^\circ - \cos^{-1}\left(\frac{98}{16\sqrt{2}}\right)$