MAT# 311 — Homework #10 — SOLUTIONS

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 # 1(a) Compute grad $f = \nabla f$ for $f = \sin x + e^{xy} + z$

$$\frac{\partial f}{\partial x} = \cos x + ye^{xy}$$

$$\frac{\partial f}{\partial y} = xe^{xy}$$

$$\frac{\partial f}{\partial z} = 1$$

$$\therefore \boxed{\text{grad } f = \left(\cos x + ye^{xy}, xe^{xy}, 1\right)}$$

 # 9 $f(x,y,z) = x + xyz$ find $D_u f$ at point $(1,-2,2)$

(a) for $u \parallel 2i + 2j - k \Rightarrow |u| = \sqrt{9} = 3$

(b) for $u \parallel 2i + 2j + k \Rightarrow |u| = \sqrt{9} = 3$

(a) $u = \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$ and $D_u f = \nabla f(1,-2,2) \cdot u$

$\nabla f = (1 + yz, xz, xy)$, $\nabla f(1,-2,2) = (-3, 2, -2)$

$D_u f = (-3, 2, -2)\left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right) = -2 + \frac{4}{3} + \frac{2}{3} = \boxed{0 = D_u f}$

(b) $u = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$,

$D_u f = \nabla f(1,-2,2) \cdot u = (-3, 2, -2) \cdot \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) =$

$D_u f = -2 + \frac{4}{3} - \frac{2}{3} = \frac{-6+4-2}{3} = \boxed{-\frac{4}{3} = D_u f}$

#12 is in next page

#14 Consider $f(x,y,z) = x^2 + yz$, Then $f(2,1,1) = 5$

$\underbrace{\nabla f(2,1,1)}_{\text{normal to the surface!!}} \perp$ level surface $f(x,y,z) = f(2,1,1)$

that is $x^2 + yz = 5$

$\nabla f = (2x, z, y)$ $\vec{n} = \nabla f(2,1,1) = \boxed{(4,1,1) = \vec{n}}$

Any scalar multiple of $(4,1,1)$ is a normal to the surface.

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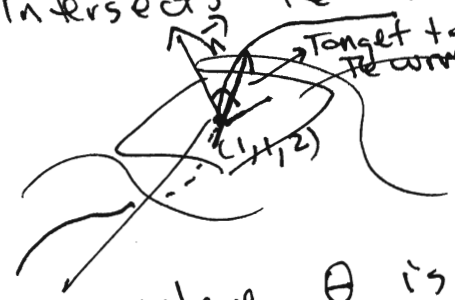
#12 Direction of maximal increase of $f(x,y,z) = e^{-xy}\cos z$ at $(1,1,0)$

ANS $\boxed{\nabla f\,(1,1,0)}$

$$\nabla f = \left(-y e^{-xy}\cos z,\ -x e^{-xy}\cos z,\ -e^{-xy}\sin z\right)$$

$$\boxed{\nabla f\,(1,1,0) = (-e, -e, 0)}$$

#27 At what angle does the curve $x = t,\ y = 2t - t^2,\ z = 2t^4$ intersects the surface $x^2 + y^3 + 3z^2 = 14$ at point $(1,1,2)$?



Tangent plane to the surface with normal $\vec{n}$.

We want the angle $\boxed{90^\circ - \theta}$

where θ is the angle between $\vec{n}$ and the tangent to the curve at the point $(1,1,2) = R(1)$ $t=1$

As in exercise 14, consider $f(x,y,z) = x^2 + y^3 + 3z^2$

Then $f(1,1,2) = 1 + 1 + 12 = 14$

So the surface is the level surface of f passing through $(1,1,2)$. Can choose

$$\vec{n} = \nabla f\,(1,1,2) = \boxed{(2, 3, 12) = \vec{n}}$$

$$|\vec{n}| = \sqrt{4+9+144} = \sqrt{157}$$

$$\nabla f = (2x, 3y^2, 6z)$$

Tangent to the curve is $R'(t) = (1, 2-2t, 8t)$

$R(1) = (1,1,2)$ $\boxed{R'(1) = (1, 0, 8)}$ $|R'(1)| = \sqrt{65}$

$$\cos\theta = \frac{\vec{n}\cdot R'(1)}{|n|\cdot|R'(1)|} = \frac{2+0+96}{\sqrt{157}\sqrt{65}} = \frac{98}{\sqrt{157}\cdot\sqrt{65}}$$

ANS $= 90^\circ - \cos^{-1}\left(\frac{98}{\sqrt{157}\sqrt{65}}\right)$