Problems in Schaum’s Book: (p. 133-134) 61*, 71*, 72*

61. Show that Green’s second identity can be written

\[
\iiint_V \left( \phi \nabla^2 \psi - \psi \nabla^2 \phi \right) \, dV = \iint_S \left( \phi \frac{d\psi}{dn} - \psi \frac{d\phi}{dn} \right) \, dS.
\]

71. Prove

\[
\iiint_V \nabla \phi \cdot \vec{A} \, dV = \iint_S \phi \vec{A} \cdot \vec{n} \, dS - \iiint_V \phi \nabla \cdot \vec{A} \, dV.
\]

72. Let \( \vec{r} \) be the position vector of any point relative to an origin \( \vec{O} \). Suppose \( \phi \) has continuous derivatives of order two, at least, and let \( S \) be a closed surface bounding a volume \( V \). Denote \( \phi \) at \( \vec{O} \) by \( \phi_0 \). Show that

\[
\oiint_S \left[ \frac{1}{r} \nabla \phi - \phi \nabla \left( \frac{1}{r} \right) \right] \cdot d\vec{S} = \iiint_V \frac{\nabla^2 \phi}{r} \, dV + \alpha,
\]

where \( \alpha = 0 \) or \( 4\pi \phi_0 \) according as \( \vec{O} \) is outside or inside \( V \). Remember \( r = |\vec{r}| \).