REVIEW # 3 - MATH 311 - FALL 2005
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The topics that we have not evaluated are:

- Volume integrals
- Integration theorems
  - Fundamental Theorem of Calculus for conservative fields
  - Divergence Theorem
  - Stoke’s Theorem
  - Green’s Theorem
- Potential Theory
  - Green’s Formulas
  - Decomposition of a field into a sum of a solenoidal field plus an irrotational field.

In the final most emphasis will be placed in the Integration Theorems. To be able to solve these problems you have to know how to compute line integrals, surface integrals and volume integrals, as well as to know how to take shortcuts using the integration theorems. Some integrals are best calculated in polar, cylindrical or spherical coordinates.

Below you have a list of problems to practice. This is much longer than the final will be. Solutions will be posted in the course webpage.

Do the following problems from our text book:

- p. 264 - # 23, # 28, # 34 (word problems)
- p. 263 - # 9, # 26 (verify integration thms)
- p. 264 - # 33 (use the results in 31, 32);
  - p. 299 - # 5, # 6, # 7
  (limits of averages of certain functions as surfaces or volumes shrink to a point)

From Schaum’s book p. 132 - # 37, # 45 and # 50. Here they are,

37. Verify Green’s theorem in the plane for

\[ \oint_C (3x^2 - 8y^2) \, dx + (4y - 6xy) \, dy, \]

where \( C \) is the boundary of the region defined by: (a) \( y = \sqrt{x}, \, y = x^2 \); (b) \( x = 0, \, y = 0, \, x + y = 1 \).

45. Show that in polar coordinates \((r, \theta)\) the expression \( x \, dy - y \, dx = r^2 \, d\theta \). Interpret

\[ \frac{1}{2} \int x \, dy - y \, dx. \]
50. Evaluate

\[ \int_{(0,1)}^{(-1,0)} \frac{-y \, dx + x \, dy}{x^2 + y^2} \]

along the following paths:

(a) straight line segments from \((1, 0)\) to \((1, 1)\) then to \((-1, 1)\), then to \((-1, 0)\).

(b) straight line segments from \((1, 0)\) to \((1, -1)\), then to \((-1, -1)\), then to \((-1, 0)\). Show that although

\[ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \]

the line integral is dependent on the path joining \((1, 0)\) to \((-1, 0)\) and explain why this doesn't contradict Green's theorem.