

MATH 311 – Fall 2005 – Exam #1 (to be used in the exam)

Table 1.2 Vector products

Concept	Geometrical Formula	Analytical Formula
		
Scalar product, dot product $\mathbf{A} \cdot \mathbf{B}$	$ \mathbf{A} \mathbf{B} \cos \theta$	$A_1 B_1 + A_2 B_2 + A_3 B_3$
Vector product, cross product $\mathbf{A} \times \mathbf{B}$	$ \mathbf{A} \mathbf{B} \sin \theta \mathbf{n}$	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$
Triple scalar product, volume [$\mathbf{A}, \mathbf{B}, \mathbf{C}$]	$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$	$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$
Parallel-perpendicular decomposition	\mathbf{B}_{\parallel}	$\frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{A} \cdot \mathbf{A}} \mathbf{A}$
	\mathbf{B}_{\perp}	$\frac{(\mathbf{A} \times \mathbf{B}) \times \mathbf{A}}{\mathbf{A} \cdot \mathbf{A}} \text{ or } \mathbf{B} - \mathbf{B}_{\parallel}$

Table 2.1 Formulas for Curves

Parametrization	$\mathbf{R} = \mathbf{R}(t)$
Velocity	$\mathbf{v} = \frac{d\mathbf{R}}{dt}$
Arc length	$ds = \mathbf{d}\mathbf{R} = \mathbf{v} dt$
Acceleration	$\begin{aligned} \mathbf{a} &= \frac{d^2\mathbf{R}}{dt^2} = \frac{d^2s}{dt^2} \mathbf{T} + k \left(\frac{ds}{dt} \right)^2 \mathbf{N} \\ &= \frac{d \mathbf{v} }{dt} \mathbf{T} + k \mathbf{v} ^2 \mathbf{N} \end{aligned}$
Curvature	$k = \left \frac{d\mathbf{T}}{ds} \right $
Radius of curvature	$\rho = \frac{1}{k}$
Tangent	$\mathbf{T} = \frac{\mathbf{v}}{ \mathbf{v} }$
Normal	$\mathbf{N} = \frac{1}{k} \frac{d\mathbf{T}}{ds}$
Binormal	$\mathbf{B} = \mathbf{T} \times \mathbf{N}$
Torsion	$\tau = -\mathbf{N} \cdot \frac{d\mathbf{B}}{ds} = (\pm) \left \frac{d\mathbf{B}}{ds} \right $
Frenet formulas	$\begin{aligned} \frac{d\mathbf{T}}{ds} &= k\mathbf{N} \\ \frac{d\mathbf{N}}{ds} &= -k\mathbf{T} + \tau\mathbf{B} \\ \frac{d\mathbf{B}}{ds} &= -\tau\mathbf{N} \end{aligned}$
(Chain rule)	$\left(\frac{d}{ds} = \frac{1}{ \mathbf{v} } \frac{d}{dt} \right)$

1.14 Vector Identities

Of the following identities, the first is the most important because the other three can be derived from it fairly easily:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C} \quad (1.30)$$

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{B} \cdot \mathbf{C})\mathbf{A} \quad (1.31)$$

$$(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = [\mathbf{A}, \mathbf{C}, \mathbf{D}] \mathbf{B} - [\mathbf{B}, \mathbf{C}, \mathbf{D}] \mathbf{A} \quad (1.32)$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}) \quad (1.33)$$

Table 3.1 Vector Operators

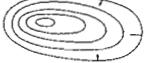
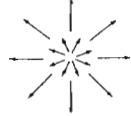
$\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$	
Name; Symbol	Interpretation
$\text{grad } \phi = \nabla \phi$	Maximum rate of change of ϕ , in the maximal direction
	
$\text{div } \mathbf{F} = \nabla \cdot \mathbf{F}$	Net outflux of \mathbf{F} per unit volume
	
$\text{curl } \mathbf{F} = \nabla \times \mathbf{F}$	Swirl of \mathbf{F} per unit area
	
Laplacian $\phi = \nabla^2 \phi = \Delta \phi$	A measure of difference between $\phi(\mathbf{R})$ and the average of ϕ around \mathbf{R}

Table 3.2 Vector Operator Identities

\mathbf{F} and \mathbf{G} denote vector fields, ϕ denotes a scalar field, and $\mathbf{R} = xi + yj + zk$. \mathbf{A} is any constant vector, and f is any differentiable function of a single variable.

$\nabla(\phi_1 \phi_2) = \phi_1 \nabla \phi_2 + \phi_2 \nabla \phi_1$	(3.27)
$\nabla \cdot \phi \mathbf{F} = \phi \nabla \cdot \mathbf{F} + \mathbf{F} \cdot \nabla \phi$	(3.28)
$\nabla \times \phi \mathbf{F} = \phi \nabla \times \mathbf{F} + \nabla \phi \times \mathbf{F}$	(3.29)
$\nabla f(\phi) = \frac{df}{d\phi} \nabla \phi$	(3.30)
$\nabla \cdot (\mathbf{R} - \mathbf{A}) = 3$	(3.31)
$\nabla \times (\mathbf{R} - \mathbf{A}) = \mathbf{0}$	(3.32)
$\nabla(\mathbf{R} - \mathbf{A} ^n) = n \mathbf{R} - \mathbf{A} ^{n-2}(\mathbf{R} - \mathbf{A})$	(3.33)
$\mathbf{F} \cdot \nabla(\mathbf{R} - \mathbf{A}) = \mathbf{F}$	(3.34)
$\nabla(\mathbf{A} \cdot \mathbf{R}) = \mathbf{A}$	(3.35)
$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$	(3.36)
$\nabla \times (\mathbf{F} \times \mathbf{G}) = (\mathbf{G} \cdot \nabla) \mathbf{F} - (\mathbf{F} \cdot \nabla) \mathbf{G} + (\nabla \cdot \mathbf{G}) \mathbf{F} - (\nabla \cdot \mathbf{F}) \mathbf{G}$	(3.37)
$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$	(3.38)
$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla) \mathbf{G} + (\mathbf{G} \cdot \nabla) \mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$	(3.39)
$\nabla \times \nabla(\phi) = \mathbf{0}$	(3.40)
$\nabla \cdot (\nabla \times \mathbf{F}) = 0$	(3.41)
$\nabla \cdot (\nabla \phi_1 \times \nabla \phi_2) = 0$	(3.42)