The linear algebra of quantum mechanics

- \( \mathcal{H} \) = set of states \( \psi \) of a physical system \( S \); superposition axiom: \( \mathcal{H} \) is a \( C^* \)-space.

- \( \mathcal{A} \) = set of measurable quantities (observables); eg: positions, momenta, etc.

- For each \( A \in \mathcal{A} \) one has a set \( \sigma(A) \subset \mathbb{R} \), the set of all possible values of \( A \).

- For each \( A \in \mathcal{A} \) one has a set \( \mathcal{H}_A \subset \mathcal{H} \), the set of pure states for \( A \).

- If \( S \) is in state \( \psi^A \) and a measurement of \( A \) is performed then the value of the measurement is \( \lambda^A \) & \( S \) moves to a state equv to \( \psi^A \) (i.e. \( \in \mathcal{C} \cdot \psi^A \)).

- If \( S \) is in a state \( \psi = \sum a_n \psi^A_n \) (where \( \in \mathcal{C} \), \( \psi \) pure) & a meas of \( A \) is performed then the value of meas is one of the \( \lambda^A \), \( S \) moves to a state equv to \( \psi \).

The probability that this happens for an \( n \) is \( a_n^2 \) provided \( \psi \) is normalized.

(i.e. \( \sum |a_n|^2 = 1 \)). The prob is \( p = |a_n|^2 \). The average value of \( A \) (for state \( \psi \)) is \( \sum |a_n|^2 \lambda^A = \sum p_n \lambda^A_n \).

- Clearly \( \langle \psi \| \psi \rangle \) linear in 1st arg; also \( \langle \psi^A_n \| \psi^A_m \rangle = 0 \) for \( m \neq n \) & \( \delta = 1 \) for \( m = n \).

- Axiom: \( \langle \psi \| \psi \rangle \) is Hermitian inner prod. (so \( \{ \psi^A_n \}_n \) orthonormal basis).

- Next aim: justify why "\( A \) acts on \( \mathcal{H} \)" is reasonable. Indeed: \( \forall \psi \) I define \( A \)(\( \sum a_n \psi^A_n \)) = \( \sum a_n \psi^A_n \).

- Computing prob that system \( S \) goes from \( \psi^B \) to \( \psi^A \) after measuring first \( B \) & then \( A \) & \( \forall \psi^A \in \mathcal{H} \), \( \sum \text{prob}(\psi^B \rightarrow \psi^B) \cdot \text{prob}(\psi^B \rightarrow \psi^A) = \sum \langle \psi^C \| \psi^B \rangle \langle \psi^B \| \psi^A \rangle \).

So if \( U = (\langle \psi^C \| \psi^B \rangle) \)

then \( U_A = U \circ U \) \( \circ \) \( U_B \).