\[ \frac{\phi(n)}{n} \leq \frac{6}{\pi^2} \]

- \( \text{If } n \text{ is a prime power, } a \text{ and } b \text{ are relatively prime, } \frac{\phi(a \cdot b)}{ab} = \frac{\phi(a)}{a} \cdot \frac{\phi(b)}{b} \)

- For any prime power \( p^k \), the number of positive integers relatively prime to it is given by \( \phi(p^k) = p^k - p^{k-1} \)

- If \( n \) is the product of distinct primes, \( \phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right) \)

- If \( a \) and \( b \) are relatively prime, \( \phi(ab) = \phi(a) \phi(b) \)

- The value of \( \phi(n) \) for a given integer \( n \) can be found by expressing \( n \) as a product of distinct prime powers and using the formula above.

- The function \( \phi(n) \) is multiplicative, meaning that if \( n = ab \) where \( \gcd(a, b) = 1 \), then \( \phi(n) = \phi(a) \phi(b) \).

- The number of positive integers less than or equal to \( n \) and relatively prime to \( n \) is given by \( \phi(n) \).

- The sum of the \( \phi \) function over all divisors of \( n \) is equal to \( n \).

- The \( \phi \) function counts the number of integers up to \( n \) that are relatively prime to \( n \).

- The \( \phi \) function is also known as the Euler's totient function.
LEGENDRE SYMBOL FOR $p > 2$, EULER'S CRIT. $p = 2^a, 6^2$

- Constr of $\mathbb{F}_p^*$, $p \geq 3$ via $x^2 - d = f(x)$, $(\frac{d}{p}) = -1$, hence.
- Constr of $\mathbb{F}_4 \subset \mathbb{F}_8 \subset \mathbb{F}_{16}$.
- Any elt in $\mathbb{F}_p^*$ is a sum of $2^n$ squares.
- $p \geq 3 \Rightarrow (\mathbb{Z}/p\mathbb{Z})^* \text{ cyclic}$, (not true for $p = 2, n = 3$)
- (Artin, ex. 3, p. 296) $\sigma \in \mathbf{G} \setminus \mathbf{Q}$, $E \in \mathbf{C}$ w.r.t. $\sigma \neq E$. Then $A$ fin. dim. ext. $\mathbf{F}_\sigma$ is cyclic.
- $\sigma \in G(\mathbf{Q}/\mathbf{Q})$; then $A$ fin. ext. $\mathbf{F}_\sigma$ is cyclic.
- $A$ # field contains only fin. many roots of $1$. [if not $\Phi(k(x))k$
- $K = \mathbf{C}$, $F/k$ field ext. of fields only fin. many fields.
- $\text{disc}(K) = \mathbf{Q}$, $\sigma : k \to k^\sigma$, $\sigma(x) = x + 1$. (always $k = 0$)
- $S = \sum a \sigma$ is finite. Prove $S^2 = p (\frac{-1}{p}) = (1 - \frac{1}{p}) = \alpha P$, $S = \mathbf{F}(\mathbf{F}_p) \subset \mathbf{C}(\mathbf{F}_p, \alpha)$

**May ass. $F/E$ normal. Then $\sigma \in G(F/E)$ & $F \langle \sigma \rangle = E$ so $E \subset G(F/E)$**

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**May ass. $F/E$ normal. Then $\sigma \in G(F/E)$ if $G = H_0$**

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\begin{align*}
\text{Proof} & \text{of irreducibility. Use}\ $F \subset \mathbf{C}$, use $F$ an ext. of $\mathbf{Q}$ w.r.t. $\sigma$. Set $F_i = F_1(x) / (x^2 + x + 1)$
\end{align*}
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