Math 561, Fall 2018
Assignment 8, due Wednesday, October 24

Hand in solutions to the following exercises:

1. Greene & Krantz, Chapter 3, Exercise 37.
2. Greene & Krantz, Chapter 3, Exercise 40.
3. Greene & Krantz, Chapter 4, Exercise 3.

   Note: There is of course an implicit assumption that f does not vanish on $U \setminus \{P\}$.
4. Greene & Krantz, Chapter 4, Exercise 5.

   Note: Don't forget the part (g) on p. 146! Also, make sure to justify your answers.
5. Let $f(z)$ be an analytic function on $\{z : 0 < |z| < 2\}$ such that $f((-1)^n/n) = (-1)^n$ for all positive integers n. Prove that
$$\inf_{0<|z|<2} |f(z)| = 0.$$

On your own: Greene & Krantz: Chapter 3, Exercises 42, 49, 60[1]. Chapter 4, Exercises 1, 4, 8a. Also the following problem:

1. Let U be a connected open set, $P \in U$, and f be a nonconstant analytic function on $U \setminus \{P\}$. Suppose that f takes on a certain value α at least once on any punctured disk $D(P, r) \setminus \{P\}$ (i.e. $\alpha \in f(D(P, r) \setminus \{P\})$ for each $r > 0$ with $D(P, r) \subset U$). Prove that f has an essential singularity at P.
2. Let $A = \sum_{n=0}^{\infty} a_n$, $B = \sum_{n=0}^{\infty} b_n$ be convergent, complex series and define $c_n = \sum_{k=0}^{n} a_k b_{n-k}$. Consult a real analysis book to find sufficient conditions (and a proof!) which ensure that $\sum_{n=0}^{\infty} c_n = AB$. Then on your own, use this to prove that if e^z is defined by $\sum_{k=0}^{\infty} \frac{z^k}{k!}$, then $e^{w+z} = e^w e^z$ for any $w, z \in \mathbb{C}$.

Reading: Greene & Krantz: finish Chapter 3, start Chapter 4.

---

[1]This is admittedly a nonessential problem for what we are concerned with in this class. This exercise is better suited for a course in real analysis such as Math 510. Nonetheless, you should at least contemplate why this means that differentiability in the complex sense yields much different properties in contrast to what can be said about C^∞ functions on the real line.