Math 561, Fall 2018
Assignment 5, due Wednesday, September 26

Hand in solutions to the following exercises:

1. Greene & Krantz, Chapter 2, Exercise #32.

   Note: The Green's theorem argument works here, but it makes the problem a little too easy. Instead take this as an opportunity to explore the contour deformations from class and §2.6 in the text.

2. Greene & Krantz, Chapter 2, Exercise #36.

3. Greene & Krantz, Chapter 2, Exercise #40.

4. (a) Suppose that a curve C is the boundary of a domain $\Omega \subset \mathbb{C}$. The standard conclusion in Green's theorem reads as $\oint_C u\,dx + v\,dy = \iint_\Omega v_x - u_y\,dxdy$. Show that this in turn implies that
   $$\oint_C F\,dz = 2i \iint_\Omega \frac{\partial F}{\partial \bar{z}}\,dxdy.$$
   In other words, show that the identity $(*)$ on p. 490 of the text implies the identity $(**)$ on the same page.

   (b) Greene & Krantz, Chapter 2, Exercise #44.
   Hint: Argue that for any $z_0 \in U$,
   $$\lim_{r \to 0+} \frac{1}{\pi r^2} \iint_{D(z_0,r)} \frac{\partial F}{\partial \bar{z}}\,dxdy = \frac{\partial F}{\partial \bar{z}}(z_0).$$
   You don't really need to involve uniform continuity here as in the previous assignment, it should be sufficient just to use continuity of $\frac{\partial F}{\partial \bar{z}}(z)$ at z_0.

On your own: Greene & Krantz: Chapter 2, Exercises 24, 26, 37. Chapter 3, Exercises 2, 5(b,c)[1].

Reading: Greene & Krantz 3.1-3.2.

[1]We proved part (a) in class during our discussion of connectedness so there's no need to prove it again.