Math 561, Fall 2018
Assignment 2, due Wednesday, September 5

Hand in solutions to the following exercises. You may use the chain rule in several variables as much as you like, in particular it is important for Exercises 3 and 4.

1. Suppose that $U \subset \mathbb{R}^2$ is open and connected. In 1b, consider U to be a subset of $\mathbb{C}$.
   (a) Suppose that $f : U \to \mathbb{R}$ has first partial derivatives satisfying $\frac{\partial f}{\partial x}(x,y) = 0$ and $\frac{\partial f}{\partial y}(x,y) = 0$ for all $(x,y) \in U$. Prove that $f(x,y)$ is constant on U.
   Note: Use the result from class that if $U \subset \mathbb{R}^2$ is open and connected, then any two points in U can be joined by a polygonal path consisting of line segments parallel to the coordinate axes.
   (b) Suppose that $f : U \to \mathbb{C}$ satisfies $\frac{\partial f}{\partial z} = 0$ and $\frac{\partial f}{\partial \bar{z}} = 0$ for all $z \in U$. Prove that f is constant on U.
2. Greene & Krantz, Chapter 1, Exercise #33.
3. Greene & Krantz, Chapter 1, Exercise #36.
4. Greene & Krantz, Chapter 1, Exercise #49.

On your own (i.e. do not hand these in for a grade):

Greene & Krantz Ch. 1, Exercises 23, 26, 27, 28, 30[1], 34, 41, 46 and the following problem:

1. (Better late than never) Let $z \in \mathbb{C}$. Prove the following identities.
   (a) $\mathrm{Re}\,(z) = \frac{1}{2}(z + \bar{z})$.
   (b) $\mathrm{Im}\,(z) = \frac{1}{2i}(z - \bar{z})$.
   (c) $\mathrm{Re}\,(iz) = -\mathrm{Im}\,(z)$.
   (d) $\mathrm{Im}\,(iz) = \mathrm{Re}\,(z)$.

Reading: Finish your review of topology, at least have the two sections from Taylor's book read, as well as Conway's treatment of connected sets. Also Greene & Krantz 1.3-1.5.

[1] Find a proof which does not appeal to Exercise 1 above, rather just deduce that each coefficient of the polynomial vanishes.