Math 402/502, Spring 2020
Assignment 7, due Thursday, March 26

Problems to hand in:

1. Wade, Exercise 10.1.6.
2. Wade, Exercise 10.1.7.
3. Wade, Exercise 10.2.2, but add the following as a "part (c)" to the problem:
   (c) Let (X, ρ), (Y, τ) be metric spaces. Suppose $f : X \to Y$ is any function. Prove that if a is an isolated point in X, then f is continuous at a.
4. Wade, Exercise 10.2.3.

On your own: Wade 10.1.4, 10.1.8[1], 10.1.12[2], 10.2.4, 10.2.5, 10.2.6, as well as the following problem:

1. Let X be a set and suppose ρ, τ are two metrics on X. A priori, nothing says that the collections of open sets defined by these two metrics will be the same. For example, consider $\mathbb{R}$ with the usual metric and the discrete metric, the latter has *many* more open sets (why?).

   However, the metrics ρ, τ are said to be *equivalent* if there are $C_1, C_2 > 0$ such that

   $$C_1\rho(x, y) \leq \tau(x, y) \leq C_2\rho(x, y) \quad \text{for any } x, y \in X.$$

   In this case, prove that a set $V \subset X$ is open with respect to ρ if and only if it is open with respect to τ. Hence the two metrics generate the same open sets.

Reading: Finish sections 10.1 and 10.2 of the Wade text. Start section 10.3.

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[1] Part (a) follows very easily from Lemma 7.11.

[2] Exercises like this are about as good of a review problem as you can find. Revisit the proofs of these statements for real-valued sequences (as in Math 401/501), then observe how they easily generalize to metric spaces.