Math 401/501, Fall 2018
Assignment 5 and some notes for the first midterm
Not Collected

Exercises (do not hand these in, but have them completed prior to Sept. 26):

1. Ross, §7 Exercises: 7.1, 7.3, 7.5.
2. Ross, §8 Exercises: 8.1, 8.2, 8.3, 8.4, 8.5, 8.7.
3. Suppose $A \subset \mathbb{R}$ is a set which is bounded from above and below. Prove that
$$\sup A - \inf A = \sup\{a_1 - a_2 : a_1, a_2 \in A\}.$$

Reading: Ross, §7-8.

Some notes concerning the first midterm:

1. The first midterm is on Wednesday, September 26 in class. **Bring your UNM photo ID.**
2. The first midterm will cover everything we have discussed since the beginning of class. This includes all of the Hammack text, *except* for Chapters 3, 11, 13. It also will cover the following sections from Ross: §1, 2, 3, 4, 5, 7, 8.
3. There is no practice exam for the midterm, but recall that there are numerous "on your own" exercises in each assignment. If you haven't been diligent about them, now is the time to revisit them in detail.
4. There will be student-driven reviews in class on Monday, September 24 and in recitation on Tuesday, September 25. You should come prepared to ask any questions you have.
5. It is unlikely that problems along the lines of the §7 exercises above will appear on the midterm: view them as exercises designed to familiarize yourself with sequences. However, problems along the lines of the §8 exercises may appear on the exam. In particular, you may be asked to provide a detailed ϵ, N proof that a certain limit exists. At this stage, we are still looking to see that you can construct such a proof rather than relying on some of the limit properties concerning sums, products, and quotients that will be established in §9.
6. The exam also may have problems concerning the supremum or infimum of a set. Recall that the supremum of a nonempty set $S \subset \mathbb{R}$ which is bounded above, denoted as $\sup S$ is characterized by two properties, giving the quantity its meaning as the "least upper bound":

   (i) $\sup S$ is an upper bound for S.

   (ii) If M is any upper bound for S, then $\sup S \leq M$.

   As discussed in class, (ii) is logically equivalent to its contrapositive:

   (ii)′ If $y < \sup S$, then y is not an upper bound for S, that is, there exists $x \in S$ such that $y < x$.

   Consequently, if you want to prove that a certain value is the supremum of a set, you should prove (i) and either (ii) or (ii)′ (either one is fine since they are both logically equivalent). In the other direction, if you are given $\sup S$ and seek to deduce something from this, you can use that $\sup S$ satisfies (i), (ii), and (ii)′ to this end. As noted in class, the infimum is defined similarly as the "greatest lower bound".