Math 311 Practice Exam 2

1. Compute the curl of the vector field $\mathbf{F}(r, \phi, \theta) = r\,\mathbf{e}_\phi + \mathbf{e}_\theta$ in spherical coordinates.

2. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = z^2\mathbf{i} - \sin y\mathbf{j} + (2xz + 6z^2)\mathbf{k}$.

a. Determine whether or not $\mathbf{F}$ is conservative. If it is conservative, find a potential function for $\mathbf{F}$.

b. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{R}$, where C is the path parameterized by

$$\mathbf{R}(t) = te^{1-t}\,\mathbf{i} + \frac{\pi}{2}\,t\,\mathbf{j} + t^4\,\mathbf{k}, \qquad 0 \le t \le 1.$$

Simplify your answer. (Hint: Try to use your answer from part **a**. What are the initial and terminal points of the path?)

3. Let $\mathbf{F}$ be the two dimensional vector field $\mathbf{F} = \sin x\,\mathbf{i} - y\cos x\,\mathbf{j}$.

a. Show that $\mathbf{F}$ is solenoidal.

b. Find a vector potential for $\mathbf{F}$.

4. Let S be the portion of the cone $z^2 = x^2 + y^2$ lying in the first octant with $0 \le z \le 2$ and oriented upward (so that $\mathbf{n} \cdot \mathbf{k}$ is positive). Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} + \mathbf{k}$.

5. Let V be the domain bounded by the cylinder $x^2 + y^2 = 4$ which lies below the plane $z = 2 + x$ and above the $z = 0$ plane. Find the integral of $f(x, y, z) = x^2 + y^2$ over V, that is, find

$$\iiint_V x^2 + y^2\, dV.$$

Show your work.