Math 311–Practice exam 1

1. Write the vector \( \mathbf{A} = 2\mathbf{i} + \frac{1}{2}\mathbf{j} \) as the sum of vectors \( \mathbf{A} = \mathbf{B} + \mathbf{C} \) where \( \mathbf{B} \) is parallel to \( \mathbf{D} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \) and \( \mathbf{C} \) is perpendicular to \( \mathbf{D} \).

2 a. A curve is parameterized by the vector function

\[
\mathbf{R}(t) = 2(t+2)\mathbf{i} + e^{2t}\mathbf{j} + (2 - 4t + t^2)\mathbf{k}.
\]

Find a unit tangent vector to the curve at the point \((4, 1, 2)\).

b. Find the tangent plane to the surface \(\frac{1}{4}x^2 + y^2 = z^2 + 1\) at the point \((4, 1, 2)\).

c. The curve from part a) and the surface from part b) intersect at the point \((4, 1, 2)\). What is the angle between the curve and the surface at this point? Explain your answer.

3. A particle moves in a circular path with nonconstant angular velocity, for times \(0 \leq t \leq \sqrt{2\pi}\). Its position \( \mathbf{R}(t) \) at time \( t \) is given by

\[
\mathbf{R}(t) = \frac{1}{4}\cos(t^2)\mathbf{i} + \frac{1}{4}\sin(t^2)\mathbf{j}.
\]

Note that \(\cos(t^2)\) means that the cosine function is evaluated at \(t^2\).

a. (5 points) Find the speed of the particle as a function of time.

b. (10 points) Compute the \( a_t \), the tangential component of acceleration at each time.

c. (10 points) What is the curvature of the path of the particle? Explain.

4. Suppose \( \mathbf{A} \) is constant vector and \( \phi(x, y, z) \) is a scalar function defined on all of \( \mathbb{R}^3 \). Use vector identities to show that \( \nabla \times (\phi \mathbf{A}) \) at the point \((x, y, z)\) is perpendicular to both \( \mathbf{A} \) and \( \nabla \phi(x, y, z) \).

5. True/False section: Determine if the following statements are true or false, and explain your answer. You will be graded both on your answer and your explanation. Explanations must be given for full credit.

a. (5 points) The vectors \( \mathbf{i} + 2\mathbf{j} + \mathbf{k}, -\mathbf{i} - \mathbf{j}, \text{ and } \mathbf{j} + \mathbf{k} \) are all parallel to a plane.

b. (5 points) There exists vector field \( \mathbf{F} \) whose curl is equal to \( \mathbf{G} = e^y\mathbf{i} + y\mathbf{j} + e^z\mathbf{k} \)
6. A particle is moving along a curve $C$ which has a binormal vector $B$. Is the acceleration vector of the particle always perpendicular to $B$?

7. Show that for any two fixed vectors $A$ and $B$,

$$|A \times B|^2 + (A \cdot B)^2 = |A|^2 |B|^2.$$