Math 311–Practice exam 1

1. Write the vector $\mathbf{A} = 2\mathbf{i} + \frac{1}{2}\mathbf{j}$ as the sum of vectors $\mathbf{A} = \mathbf{B} + \mathbf{C}$ where $\mathbf{B}$ is parallel to $\mathbf{D} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{C}$ is perpendicular to $\mathbf{D}$.

2 a. A curve is parameterized by the vector function

$$\mathbf{R}(t) = 2(t+2)\mathbf{i} + e^{2t}\mathbf{j} + (2 - 4t + t^2)\mathbf{k}.$$

Find a unit tangent vector to the curve at the point $(4, 1, 2)$.

b. Find the tangent plane to the surface $\frac{1}{4}x^2 + y^2 = z^2 + 1$ at the point $(4, 1, 2)$.

c. The curve from part a) and the surface from part b) intersect at the point $(4, 1, 2)$. What is the angle between the curve and the surface at this point? Explain your answer.

3. A particle moves in a circular path with *nonconstant* angular velocity, for times $0 \leq t \leq \sqrt{2\pi}$. Its position $\mathbf{R}(t)$ at time t is given by

$$\mathbf{R}(t) = \frac{1}{4}\cos(t^2)\mathbf{i} + \frac{1}{4}\sin(t^2)\mathbf{j}.$$

Note that $\cos(t^2)$ means that the cosine function is evaluated at t^2.

a. (5 points) Find the speed of the particle as a function of time.

b. (10 points) Compute the a_t, the tangential component of acceleration at each time.

c. (10 points) What is the curvature of the path of the particle? Explain.

4. Suppose $\mathbf{A}$ is constant vector and $\phi(x, y, z)$ is a scalar function defined on all of $\mathbb{R}^3$. Use vector identities to show that $\nabla \times (\phi\mathbf{A})$ at the point (x, y, z) is perpendicular to both $\mathbf{A}$ and $\nabla\phi(x, y, z)$.

5. True/False section: Determine if the following statements are true or false, and explain your answer. You will be graded both on your answer and your explanation. Explanations **must** be given for full credit.

a. (5 points) The vectors $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $-\mathbf{i} - \mathbf{j}$, and $\mathbf{j} + \mathbf{k}$ are all parallel to a plane.

b. (5 points) There exists vector field $\mathbf{F}$ whose curl is equal to $\mathbf{G} = e^y\mathbf{i} + y\mathbf{j} + e^z\mathbf{k}$

6. A particle is moving along a curve C which has a binormal vector $\mathbf{B}$. Is the acceleration vector of the particle is always perpendicular to $\mathbf{B}$?

7. Show that for any two fixed vectors $\mathbf{A}$ and $\mathbf{B}$,

$$|\mathbf{A} \times \mathbf{B}|^2 + (\mathbf{A} \cdot \mathbf{B})^2 = |\mathbf{A}|^2 |\mathbf{B}|^2.$$