Math 510
Assignment 7, due Thursday, October 24

Exercises to hand in. As usual, hand in Parts I and II separately:

Part I:

1. Rudin, Chapter 4, #18. For the sake of simplicity, restrict attention to the interval (0,1), that is, prove continuity at every irrational point in (0,1) and discontinuity at every rational point in (0,1).

2. Rudin, Chapter 4, #20.

3. Rudin, Chapter 4, #22.

Part II:

1. Rudin, Chapter 3, #8.

2. Rudin, Chapter 4, #4.

On your own: Rudin, Chapter 4: 1, 2, 3, 5, 7

Important Review for Chapter 4 (also on your own): Prove the following properties concerning a function \( f : X \to Y \), \( E, E_\alpha \subset Y \), \( G \subset X \):

(a) \( f^{-1}(E^C) = [f^{-1}(E)]^C \)
(b) \( f(f^{-1}(E)) \subset E \)
(c) \( G \subset f^{-1}(f(G)) \)
(d) \( f^{-1}(\bigcup_\alpha E_\alpha) = \bigcup_\alpha f^{-1}(E_\alpha) \)
(e) \( f^{-1}(\bigcap_\alpha E_\alpha) = \bigcap_\alpha f^{-1}(E_\alpha) \)

Properties (a), (b), (c) are used in Chapter 4, properties (d), (e) are also common. Find examples of functions \( f \) such that equality in (b) and (c) fails to hold. Prove that equality in (b) holds whenever \( f \) is surjective and equality in (c) holds whenever \( f \) is injective.

Reading: Rudin, Chapter 4