# Math 510
# Assignment 7, due Friday, October 28

Exercises to hand in:

1. Rudin, Chapter 4, #11
2. Rudin, Chapter 4, #14
3. Rudin, Chapter 4, #18
4. Let $(X, d)$ be a compact metric space and $F$ a mapping $F : X \to X$ such that
$$d(F(x), F(y)) < d(x, y) \quad \text{for all} \quad x, y \in X.$$
   (a) Show that the function $g : X \to [0, \infty)$ defined by $g(x) = d(x, F(x))$ is a continuous function whose minimum must be zero.
   (b) Show that the equation $F(x) = x$ has exactly one solution, that is, $F$ has a unique fixed point.
5. (a) Show that the function $f(x) = \sin(\frac{\pi}{x})$ is continuous on the interval $(0, 1)$.
   (b) Is $f$ uniformly continuous on $(0, 1)$?
   (c) For a real-valued function $g$ defined on a metric space $(X, d)$ let
$$w(r) = \sup\{|g(x) - g(x')| : d(x, x') \leq r\}.$$
   Show that $g$ is a uniformly continuous function if and only if
$$\lim_{r \to 0} w(r) = 0.$$

On your own: Rudin, Chapter 4: 8, 10, 15, 19, and the following problem:

Let $f : \mathbb{R} \to \mathbb{R}$ be continuous at $x = 0$, and suppose that
$$f(x + y) = f(x) + f(y) \quad \text{for all} \quad x, y \in \mathbb{R}.$$
Show that $f(x) = cx$ for some $c \in \mathbb{R}$.

Reading: Rudin, Chapter 4, start Chapter 5.