Math 510
Assignment 6, due Thursday, October 3

Hand in Parts I and II of this assignment separately:

Part I:

1. Rudin, Chapter 3, #6 a,b,d
   (For 6d, your answer will depend on $z$. In the case $|z| = 1$, assume that $z^n \neq -1$ for all $n$.)

2. Rudin, Chapter 3, #14 a,b,c

Part II:

1. Rudin, Chapter 3, #7

2. (a) Let $s_n := 1 + \frac{1}{2} + \cdots + \frac{1}{n}$. Prove that $s_{2n} - s_n > \frac{1}{2}$ and use this to show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. In particular, you may not use Theorem 3.28 to solve the problem.
   (b) Consider the following series

   $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{8} + \cdots$

   In other words, the general term is

   $$a_n = \begin{cases} \frac{1}{n} & \text{if } n = 2^k, \ k = 1, 2, 3, \cdots \\ \frac{1}{n} & \text{otherwise} \end{cases}$$

   Show that the series diverges. You are allowed to use Theorem 3.47 to solve the problem.

On your own: Rudin, Chapter 3: 5, 9, 10, 16 (a,b) and the following problem:

Suppose $\{a_n\}_{n=1}^{\infty}$ is a sequence in $\mathbb{R}$ which is unbounded above. Prove that there exists an increasing subsequence $\{a_{n_k}\}_{k=1}^{\infty}$ such that $\lim_{k \to \infty} a_{n_k} = +\infty$.

Reading: Chapter 3