Hand in the following problems:

1. Taylor, §7.3 #14
2. Taylor, §7.4 #1
3. Taylor, §7.4 #2
4. Taylor, §7.4 #4
5. Taylor, §7.4 #8

6. Decide if the following sets are compact or not compact. Prove your answer by appealing to the definition of compactness, without using the Heine-Borel theorem. Thus if the set is compact, prove directly that every open cover has a finite subcover. If the set is not compact, find an open cover with no finite subcover.

   (a) Any open ball $B_r(x)$ in $\mathbb{R}^d$.

   (b) $\mathbb{N}$ as a subset of $\mathbb{R}$

   (c) $\{(\cos \left(\frac{2\pi}{n}\right), \sin \left(\frac{2\pi}{n}\right)) \in \mathbb{R}^2 : n \in \mathbb{N}\}$ as a subset of $\mathbb{R}^2$

   (Note that the point $(1, 0)$ is in the set, so even though this set lies in the plane, it is in some sense similar to the set $\{\frac{1}{n} : n \in \mathbb{N}\} \subset \mathbb{R}$)

   (d) $\{x \in \mathbb{Q} : 0 < x < 2\}$ as a subset of $\mathbb{R}$

Not collected: 7.4: #5, #6 and the following problem:

Prove that any interval of the form $(a, b]$ with $-\infty < a < b < \infty$ is neither open nor closed.

Reading: 7.4