Math 510
Assignment 5, due Friday, October 7

Exercises to hand in:

1. Rudin, Chapter 3, #7
2. Rudin, Chapter 3, #9
3. Rudin, Chapter 3, #14 a,b
4. Rudin, Chapter 3, #23

5. (a) Let \( s_n := 1 + \frac{1}{2} + \cdots + \frac{1}{n} \). Prove that \( s_{2n} - s_n > \frac{1}{2} \) and use this to show that the series \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges. In particular, you may not use Theorem 3.28 to solve the problem.

(b) Consider the following series

\[
1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \cdots
\]

In other words, the general term is

\[
a_n = \begin{cases} 
-\frac{1}{n} & \text{if } n = 2^k, \ k = 1, 2, 3, \ldots \\
\frac{1}{n} & \text{otherwise}
\end{cases}
\]

Show that the series diverges. You are allowed to use Theorem 3.47 to solve the problem.

On your own: Rudin, Chapter 3, 1-5, 6 (a,b,d), 16 (a,b)
Reading: Chapter 3