Math 510
Assignment 4, due Wednesday, September 28
(Note the unusual due date)

Exercises to hand in:

1. Rudin, Chapter 2, #16
2. Rudin, Chapter 2, #23
3. Rudin, Chapter 2, #29
4. Consider $\mathbb{R}$ as a metric space with the usual metric $d(x, y) = |x - y|$.
   (a) Show that the set $S = [0, 1] \cap \mathbb{Q}$ is not connected in $\mathbb{R}$.
   (b) Show that the interval $[0, 1]$ is connected in $\mathbb{R}$.
5. Let A be a closed subset of $\mathbb{R}^k$ and K a compact subset of $\mathbb{R}^k$ with respect to the usual metric. The distance betweeen A and K is defined to be
$$d(A, K) := \inf\{|x - y| : \ x \in A, \ y \in K\}.$$
   (a) Show that $d(A, K) > 0$ if and only if the sets A and K are disjoint.
   (b) Is the result true if K is only assumed to be closed?

On your own:

1. Determine which of the following sets in $\mathbb{R}^2$ (with the usual metric) is compact. If any of them is not compact, find the smallest compact set (if it exists) containing the given set.
   (a) $\{(x, y) \in \mathbb{R}^2 : y = \sin(1/x) \text{ for some } x \in (0, 1)\}$
   (b) $\{(x, y) \in \mathbb{R}^2 : |xy| \leq 1\}$
2. Let (X, d) be a metric space. Let E be a nonempty subset of X. Define the distance from $x \in X$ to E by
$$\rho_E(x) = \inf_{y \in E} d(x, y).$$
   Prove that $\rho_E(x) = 0$ if and only if x belongs to $\overline{E}$ (the closure of E).

Reading: Finish Chapter 2, start Chapter 3