Math 510
Assignment 2, due Thursday, September 5

Hand in Parts I and II of this assignment separately:

Part I

1. A real number x is said to be *algebraic* if it is a root of a polynomial equation with integer coefficients

$$a_n x^n + \cdots + a_1 x + a_0 = 0 \qquad a_0, a_1, \ldots, a_n \in \mathbb{Z}.$$

   If a real number is not algebraic, it is said to be *transcendental*.

   (a) Show that the set of polynomials with integer coefficients is countable.

   (b) Show that the set of algebraic numbers is countable.

   (c) Show that the set of transcendental numbers is uncountable.

2. Rudin, Chapter 2, #8 (Assume $\mathbb{R}^2$ is equipped with the usual metric)

Part II

1. Rudin, Chapter 2, #11. For d_5 you may use that the function $p : [0, \infty) \to [0, \infty)$ defined by $p(t) = \frac{t}{1+t}$ is increasing.

2. Suppose X is a set and that for each $n \in \mathbb{N}$, $d_n(x, y)$ defines a metric on X. Define a function on $X \times X$ by

$$\rho(x, y) = \sum_{n=1}^{\infty} 2^{-n} \frac{d_n(x, y)}{1 + d_n(x, y)}.$$

   Show that ρ is a metric. (You may take for granted that $\sum_{n=1}^{\infty} 2^{-n}$ is a convergent series along with other elementary facts about series.)

On your own (some may be assigned next week): 4, 7, 10
Reading: begin Ch. 2 of Rudin