# Math 510, Fall 2015
# Assignment 2, due Friday, September 4

1. A real number $x$ is said to be *algebraic* if it is a root of a polynomial equation with integer coefficients

$$a_n x^n + \cdots + a_1 x + a_0 = 0 \qquad a_0, a_1, \ldots, a_n \in \mathbb{Z}.$$

   If a real number is not algebraic, it is said to be *transcendental*.

   (a) Show that the set of polynomials with integer coefficients is countable.

   (b) Show that the set of algebraic numbers is countable.

   (c) Show that the set of transcendental numbers is uncountable.

2. Prove that if a set $S$ contains a countable set, then it is in one-to-one correspondence with a proper subset of itself. In other words, prove that there exists a proper subset $E \subsetneqq S$ such that $S \sim E$.

3. Rudin, Chapter 2, #8 (Assume $\mathbb{R}^2$ is equipped with the usual metric)

4. Rudin, Chapter 2, #11. For $d_5$ you may use that the function $p : [0, \infty) \to [0, \infty)$ defined by $p(t) = \frac{t}{1+t}$ is increasing.

Reading: begin Ch. 2 of Rudin

On your own (some may be assigned next week): Rudin, Ch. 2: 4, 7, 10 and the following problem:

Suppose $X$ is a set and that for each $n \in \mathbb{N}$, $d_n(x, y)$ defines a metric on $X$. Define a function on $X \times X$ by

$$\rho(x, y) = \sum_{n=1}^{\infty} 2^{-n} \frac{d_n(x, y)}{1 + d_n(x, y)}.$$

Show that $\rho$ is a metric. (You may take for granted that $\sum_{n=1}^{\infty} 2^{-n}$ is a convergent series along with other elementary facts about series.)