Math 402, Spring 2014
Assignment 2, due Tuesday, February 4

Hand in the following problems:

1. Taylor, §6.2 #5
2. Taylor, §6.2 #10
3. Taylor, §6.2 #11
4. Taylor, §6.2 #12
5. Find all $p \geq 0$ such that the following series converges, proving your answer:
$$\sum_{k=1}^{\infty} \frac{1}{(k+1)\ln^p(k+1)}.$$
6. Prove the following parts of the so-called "limit comparison theorem": Suppose $\sum_{k=1}^{\infty} a_k$, $\sum_{k=1}^{\infty} b_k$ are both series with $a_k \geq 0$, $b_k > 0$ for every $k = 1, 2, 3, \ldots$ and that
$$\lim_{k\to\infty} \frac{a_k}{b_k} = L.$$
   (a) Prove that if $0 \leq L < \infty$ and $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ also converges.
   (b) Prove that if $L = \infty$ and $\sum_{k=1}^{\infty} b_k$ diverges, then $\sum_{k=1}^{\infty} a_k$ also diverges.

Notes: Even though the comparison test is in §6.1, it comes up in many places in this assignment, so don't hesitate to use it. Also, for problems involving the integral test (such as the first extra problem), you may use basic facts and techniques from calculus.

Not collected: §2.3: 11, 12 and §6.2: 1-4, 6-8

Reading: 6.2, 6.3