Math 511
Assignment 1, due Friday, February 3

Exercises to hand in:

1. Rudin, Chapter 8, #15. Notes: you may assume the identity $K_N(x) = \frac{1}{N+1} \cdot \frac{1-\cos(N+1)x}{1-\cos x}$ (you can work it out on your own for fun). Also, you may assume the result from the "On your own" qualifier problem in homework 11 from last semester.

2. Rudin, Chapter 8, #22, the first half: If α is real and $-1 < x < 1$, prove Newton's binomial theorem

$$(1+x)^\alpha = 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!} x^n.$$

3. Suppose $f(x) = \sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$ with $R > 0$. Show that if f is even (i.e. $f(x) = f(-x)$) then its power series contains only even powers of x. (On your own: what if f is odd, $f(x) = -f(-x)$?)

4. Give an example of a function f defined on an interval $(-R, R)$, $R > 0$ such that f has derivatives of all orders, but cannot be expanded in a power series about 0 (i.e. there is no power series $\sum_n c_n x^n$ which converges to $f(x)$ in a neighborhood of 0).

5. (Taylor's theorem, revisited) Show that if $f \in C^n(a,b)$ (the n-th derivative of f exists on (a,b), and is continuous), then for any $x, x_0 \in (a,b)$

$$f(x) - \sum_{k=0}^{n-1} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k = \frac{(x-x_0)^n}{(n-1)!} \int_0^1 (1-s)^{n-1} f^{(n)}(x_0 + s(x-x_0))\, ds.$$

On your own: Rudin, Chapter 8, #2, 3, 10, 13 or 14.

Reading: Rudin, Chapter 8 (except for the Gamma function section).