Exercises to hand in:

1. Rudin, Chapter 6, #9

2. Rudin, Chapter 6, #17

3. Let \( f : [0, \infty) \to \mathbb{R} \) be a decreasing function. Assume that
\[
\int_0^\infty f(x) \, dx < \infty.
\]
Show that
\[
\lim_{x \to \infty} x f(x) = 0.
\]

4. Let \([a, b]\) and \([c, d]\) be closed intervals in \(\mathbb{R}\) and let \(f(x, y)\) be a continuous real-valued function on \(\{(x, y) \in \mathbb{R}^2 : x \in [a, b], y \in [c, d]\}\). Show that the function \(g : [c, d] \to \mathbb{R}\), defined by
\[
g(y) = \int_a^b f(x, y) \, dx,
\]
for all \(y \in [c, d]\),
is continuous. First you need to explain why the function \(g\) is well-defined.

5. Let \( f : [0, 1] \to \mathbb{R} \) be an increasing function. Show that for all \(x \in (0, 1]\),
\[
\frac{1}{x} \int_0^x f(t) \, dt \leq \int_0^1 f(t) \, dt.
\]

On your own: Rudin, Chapter 6, #15, and the following problem:
Given a Riemann integrable function \(f\) on \([0, 1]\), prove the convergence of the following series and find its sum:
\[
\sum_{k=1}^{\infty} \frac{1}{2^k} \int_0^2 f \left( \frac{x}{2^k} \right) \, dx.
\]

Reading: Rudin, Chapter 6.