Math 510
Assignment 10, due Friday, November 18

Exercises to hand in:

1. Rudin, Chapter 6, #9
2. Rudin, Chapter 6, #17
3. Let $f : [0, \infty) \to \mathbb{R}$ be a decreasing function. Assume that
$$\left| \int_0^\infty f(x)\, dx \right| < \infty.$$
Show that
$$\lim_{x \to \infty} x f(x) = 0.$$
4. Let $[a, b]$ and $[c, d]$ be closed intervals in $\mathbb{R}$ and let $f(x, y)$ be a continuous real-valued function on $\{(x, y) \in \mathbb{R}^2 : x \in [a, b], y \in [c, d]\}$. Show that the function $g : [c, d] \to \mathbb{R}$, defined by
$$g(y) = \int_a^b f(x, y)\, dx, \qquad \text{for all } y \in [c, d],$$
is continuous. First you need to explain why the function g is well-defined.
5. Let $f : [0, 1] \to \mathbb{R}$ be an increasing function. Show that for all $x \in (0, 1]$,
$$\frac{1}{x} \int_0^x f(t)\, dt \le \int_0^1 f(t)\, dt.$$

On your own: Rudin, Chapter 6, #15, and the following problem:
Given a Riemann integrable function f on $[0, 1]$, prove the convergence of the following series and find its sum:
$$\sum_{k=1}^\infty \frac{1}{2^k} \int_1^2 f\left(\frac{x}{2^k}\right) dx.$$

Reading: Rudin, Chapter 6.