

UNM-PNM STATEWIDE MATHEMATICS CONTEST XXXII

NOVEMBER 13, 1999

FIRST ROUND

THREE HOURS

Problem 1: Which is larger?

(a) $A = \frac{2001}{2000^2 + 2001}$ or $B = \frac{2000}{1999^2 + 2000}$.

(b) $C = \frac{1 + 2000 + 2000^2 + \dots + 2000^{1999}}{1 + 2000 + 2000^2 + \dots + 2000^{2000}}$ or $D = \frac{1 + 2001 + 2001^2 + \dots + 2001^{1999}}{1 + 2001 + 2001^2 + \dots + 2001^{2000}}$.

Problem 2: Fernando remembers all but the last digit of Adriana's phone number, and decides to choose the last digit at random in an attempt to reach her. If he only has 70c in his pocket, what is the probability that he will dial the right number before he runs out of money? (Nowadays you need 35c to make a call in town!)

Problem 3: Among all triangles that have one side of length 7cm and another of length 10cm, there is one with largest area. Calculate that area and the length of the third side.

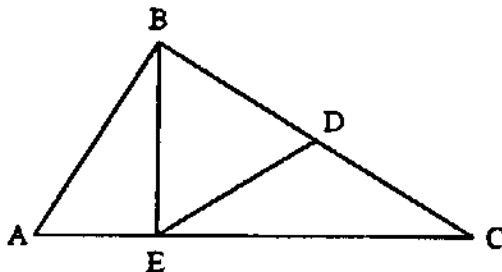
Problem 4: The integers from 1 to 2000 are written in order around a circle. Starting at 1, every fifteenth number is marked (that is: 1, 16, 31, ...). This process is continued until a number is reached which has already been marked. How many unmarked numbers remain?

Problem 5: The positive numbers $a_1 = 4$, $a_2 = 7$, $a_3 = 10, \dots$ form an arithmetic progression, that is: $a_n = a_{n-1} + r$ where r is fixed and $n = 1, 2, 3, \dots$. Calculate:

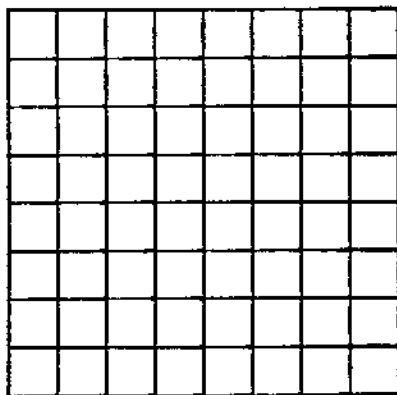
(a) a_{33}

(b) $\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{32}} + \sqrt{a_{33}}}$

Problem 6: In a triangle ABC , let D be the midpoint of BC . Assume there is a point E on AC such that the triangle BDE is equilateral with side length 4cm . Also it is known that AB is twice as long as AE . Find the area of the triangle ABC .



Problem 7: How many different rectangles consisting of an integral number of squares can be drawn on a 8×8 chessboard? (Two rectangles are considered different if they have different dimensions or different locations.)



Problem 8: Find the set E of all integers n for which

$$\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$$

is not an integer.