NEW MEXICO MATHEMATICS CONTEST XXX

NOVEMBER 15, 1997 FIRST ROUND THREE HOURS

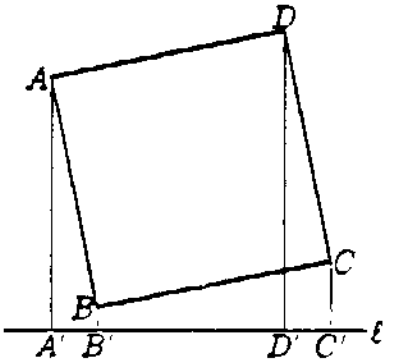
1. Find positive integers x, y, and z satisfying

$$x + \frac{y}{19} + \frac{z}{97} = \frac{1997}{19 \times 97}.$$

2. Suppose a square is projected orthogonally to a line such that the images of the two diagonals have lengths 7 and 3 cm, respectively; i.e.,

$$\overline{A'C'} = 7, \quad \overline{B'D'} = 3,$$

in the figure. What is the length of a side of the square?



3. Assume there exist coefficients p, q, r such that, for every positive integer n,

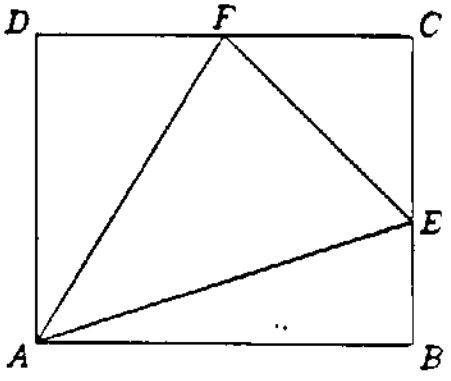
$$\frac{1^4 + 2^4 + 3^4 + \cdots + (n-1)^4 + n^4}{1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 + n^2} = pn^2 + qn + r.$$

Determine the coefficients p, q, r.

4. Suppose $\triangle AEF$ is inscribed in a rectangle $ABCD$ as in the figure. If the area of $\triangle AEF$ is 25 cm^2, and

$$\overline{BE} = 4 \text{ } cm,$$
$$\overline{DF} = 6 \text{ } cm,$$

find the area of the rectangle $ABCD$.



5. (a) Find integers a and b such that $\dfrac{4+3\sqrt{3}}{2+\sqrt{3}}$ is a root (zero) of the quadratic polynomial x^2+ax+b.

   (b) Suppose

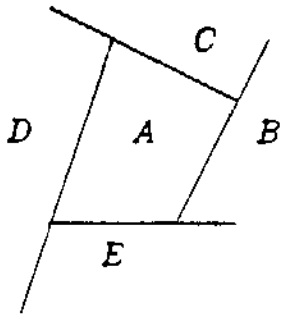
$$f(x)=x^4+2x^3-10x^2+4x-10,$$

   and

$$f\left(\frac{4+3\sqrt{3}}{2+\sqrt{3}}\right)=c\sqrt{3}+d,$$

   where c and d are integers. Find the values of c and d.

6. We want to paint all five regions in the figure such that no neighboring regions are painted by the same color.



   (a) If we have four different colors, how many ways are there to paint the regions?

   (b) What if we have five different colors?

   It is *not* necessary to use all the colors.

7. Suppose $u+v=3$, $u^2+v^2=13$.

   (a) Find the value of uv.

   (b) Find the value of u^3+v^3.

8. In $\triangle ABC$, suppose

$$\overline{AB}=5\,cm,\quad \overline{AC}=7\,cm,\quad \angle ABC=\frac{\pi}{3}.$$

   (a) Find the length of the side BC.

   (b) Find the area of $\triangle ABC$.