NEW MEXICO MATHEMATICS CONTEST XXVII

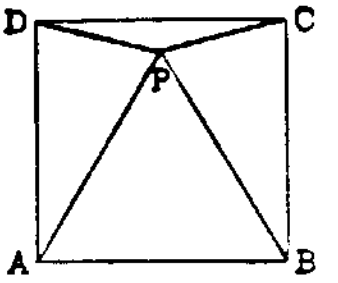
SATURDAY, NOVEMBER 13, 1993
FIRST ROUND (THREE HOURS)

1. Let a, b, c be integers satisfying

$$24a = 45b = c^2.$$

Find the smallest such positive integer c.

2. Let P be a point inside the square $ABCD$ such that $\triangle PAB$ is an equilateral triangle. Find $\angle CPD$.



3. Hester has a pocketful of pennies, nickel(s), dime(s) and quarter(s). (There is at least one coin for each type of coins.)

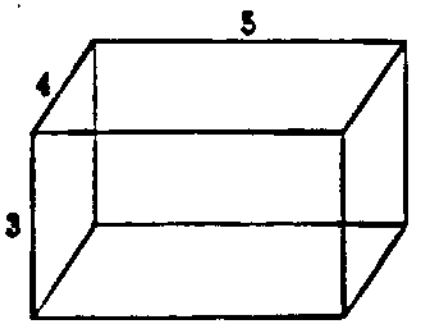
If she has a total of 19 coins worth 93 cents, how many dimes does she have ?

4. There is a box (a rectangular parallelepiped) whose height, width and length are 3, 4 and 5 (cm), respectively.

Suppose the box is sliced by a plane so that the cross section is a square.

(a) What is the maximum possible length of a side of the square ? How many distinct slices are there of this type ?

(b) What is the minimum possible length of a side of the square ? How many distinct slices are there of this type ?



5. (a) Find positive integers u and v satisfying

$$\sqrt{18-2\sqrt{65}}=\sqrt{u}-\sqrt{v}.$$

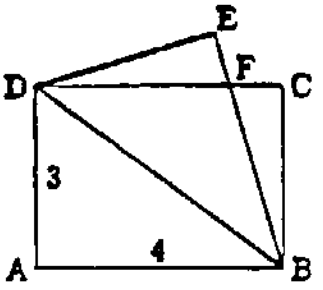
(b) Find positive integers x and y satisfying

$$\sqrt{14+3\sqrt{3+2\sqrt{5-12\sqrt{3-2\sqrt{2}}}}}=x+\sqrt{y}.$$

6. Suppose a rectangular sheet of paper $ABCD$ is folded along the diagonal BD such that A falls on E, and F is the intersection of BE and CD. If

$$\overline{AB}=4\ (cm) \quad \text{and} \quad \overline{AD}=3\ (cm),$$

then what is the area of $\triangle BDF$?



7. (a) Find the positive integers p and q (which have no common divisor other than 1) such that

$$\frac{p}{q}=0.\dot{1}8\dot{5}=0.185185185\cdots.$$

(b) Find the positive integers r and s (which have no common divisor other than 1) such that

$$\frac{r}{s}=0.1\dot{4}8\dot{6}=0.1486486486\cdots.$$

8. Let D and E be the respective midpoints of the sides AB and AC of a triangle ABC, and F is the intersection of the medians BE and CD. Suppose the area of the quadrangle $ADFE$ is 12 (cm^2), what is the area of $\triangle ABC$?