

NEW MEXICO MATHEMATICS CONTEST XXVI

NOVEMBER 14, 1992
FIRST ROUND (THREE HOURS)

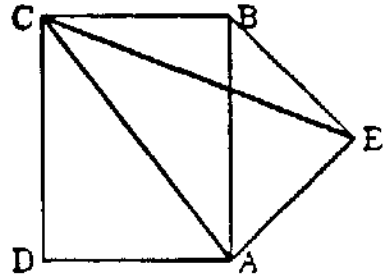
1. Evaluate

$$\frac{1992^3 - 1991 \times 1992 \times 1993}{2 \times 3 \times 83}$$

2. In the accompanying figure, $ABCD$ is a rectangle and AEB is an isosceles right triangle (i.e. $\overline{AE} = \overline{BE}$, and $\angle AEB = 90^\circ$). Suppose

$$\overline{AB} = 4 \text{ (cm)}, \quad \overline{BC} = 3 \text{ (cm)}.$$

Find the area of $\triangle AEC$.



3. Two swimmers, Sam and Jim, at the opposite ends of a 50-meter pool, start simultaneously to swim the length of the pool; Sam at the rate of $\frac{3}{4}$ of a meter per second, and Jim at $\frac{2}{3}$ of a meter per second. They swim back and forth with no loss of time at the turns.

- (a) How long will it take for Sam to catch up to Jim from behind?
- (b) How many times do Sam and Jim pass each other (swimming in the opposite direction) during this period (i.e. before Sam catches up to Jim for the first time)?

4. Suppose the incircle of $\triangle ABC$ touches the three sides BC , CA , AB at D , E , F as in the figure. If

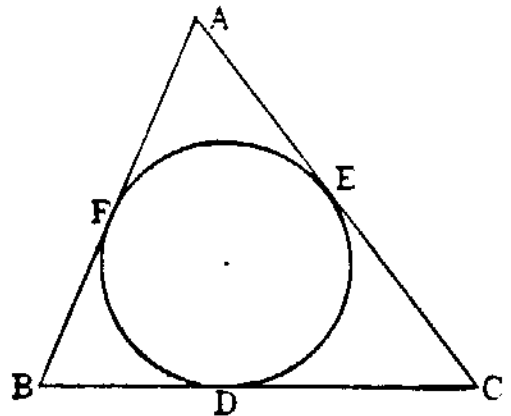
$$\overline{AB} = 13 \text{ (cm)},$$

$$\overline{BC} = 14 \text{ (cm)},$$

$$\overline{CA} = 15 \text{ (cm)},$$

then

- (a) what is the length \overline{CE} ?
- (b) what is the radius r of the incircle?



5. Find all real numbers x satisfying the inequality

$$\frac{x-1}{x-3} \geq \frac{x-2}{x-4}.$$

6. There are 30 lattice points, 5 by 6, as in the figure. Of all the squares with their 4 vertices at these lattice points,

- (a) how many have a pair of horizontal sides ?
- (b) how many have none of their sides horizontal ?

7. Let

$$f_1(x) = x, \quad f_2(x) = 1-x, \quad f_3(x) = \frac{1}{x},$$

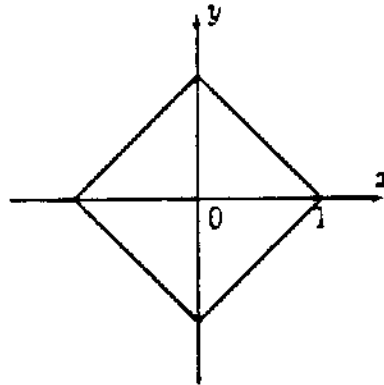
$$f_4(x) = \frac{1}{1-x}, \quad f_5(x) = \frac{x}{x-1}, \quad f_6(x) = \frac{x-1}{x}.$$

- (a) Suppose $f_6(f_m(x)) = f_4(x)$. Then $m = ?$
- (b) Suppose $f_n(f_4(x)) = f_3(x)$. Then $n = ?$

8. Note that the set of all the points (x, y) in the plane satisfying the inequality

$$|x| + |y| \leq 1$$

is a square whose area is 2 (square units).



The set of all the points (x, y) in the plane satisfying the inequality

$$\left| |x| - 1 \right| + \left| |y| - 1 \right| \leq 2$$

also forms a polygon.

- (a) How many sides has this polygon ?
- (b) What is its area ?