

NEW MEXICO MATHEMATICS CONTEST XXV

NOVEMBER 16, 1991

FIRST ROUND (THREE HOURS)

1. A *palindromic number* is an integer that reads the same forward or backward; for example, 1991 is a palindromic number. What is the difference of the two palindromic numbers closest to 1991 (but not 1991) ?
2. Lines from the vertices of a parallelogram to the midpoints of the sides are drawn as shown in Fig 1 forming a smaller parallelogram of area 7 (cm²) in the center. What is the area of the original parallelogram ?

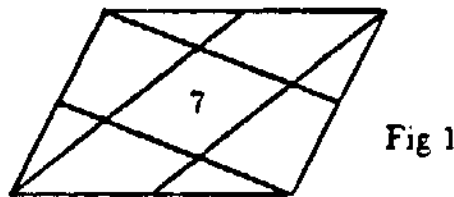


Fig 1

3. Let a, b, c and k be non-zero real numbers satisfying the relations

$$k = \frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}.$$

Find all the possible common values k of these fractions.

4. Fig 2 and Fig 3 are examples of quadrangles whose four corners can be folded to meet at a point without either overlapping or forming a gap. Of the next 6 quadrangles (Fig 4 - 9), determine which one(s) can be folded in the same manner.

Fig 4 $\overline{AB} = \overline{CD}$, $\overline{AD} = \overline{BC}$, $\overline{AB} \neq \overline{AD}$, $\angle A = 90^\circ$.

Fig 5 $\begin{cases} KN \perp KL \\ K, L, M, N \text{ are midpoints of the respective sides.} \end{cases}$

Fig 6 $\begin{cases} KM \perp LN, \overline{KM} \neq \overline{LN} \\ K, L, M, N \text{ are midpoints of the respective sides.} \end{cases}$

Fig 7 $\overline{AB} = \overline{BC}$, $\overline{AD} = \overline{CD}$.

Fig 8 $\widehat{AB} + \widehat{CD} = \widehat{BC} + \widehat{AD}$.

Fig 9 $AC \perp BD$.

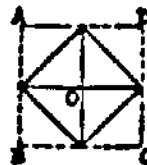


Fig 2

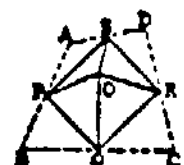


Fig 3

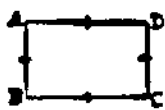


Fig 4

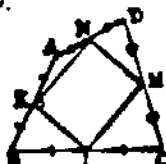


Fig 5

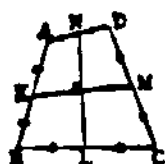


Fig 6



Fig 7



Fig 8

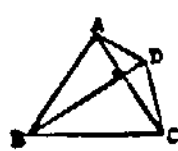


Fig 9

5. In a magic square of addition, the sum of the integers in each row, column and diagonal is the same. For example, in Fig 10, the magic sum is 15. Fill in the blanks in Fig 11 with positive integers to make it a magic square of multiplication; i.e. complete Fig 11 with positive integers so that the product of the three positive integers in each row, column and diagonal becomes the same. What is the sum of the two largest integers in the completed magic square of multiplication (Fig 11) ?

4	9	2
3	5	7
8	1	6

Fig 10

3		
4		
	1	

Fig 11

6. In Fig 12, the three line segments AB , EF , CD are perpendicular to the line BD , and the three points A , E , D are collinear (i.e. on a straight line), as are the points B , E , C and B , F , D . Suppose $\overline{AB} = 10$ (cm), $\overline{CD} = 15$ (cm). Find the length \overline{EF} .

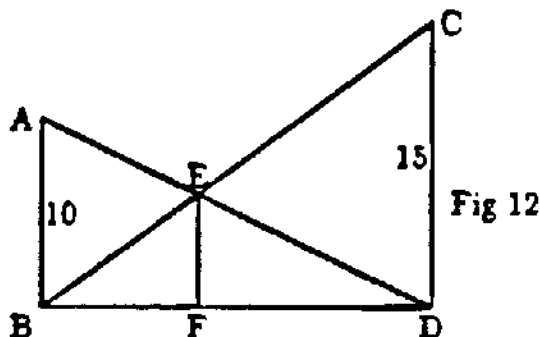


Fig 12

7. Suppose the following two quadratic equations

$$x^2 - 5x + k = 0 \quad \text{and} \quad x^2 - 9x + 3k = 0$$

have a non-zero root in common. What is the value of k ?

8. Suppose the diagonals AC and BD of a trapezoid $ABCD$ intersect at P . If the areas of $\triangle PAB$ and $\triangle PCD$ are 36 (cm^2) and 25 (cm^2), respectively, what is the area of the trapezoid $ABCD$? (Note that $AB \parallel CD$, $\angle BAD = \angle ABC$.)

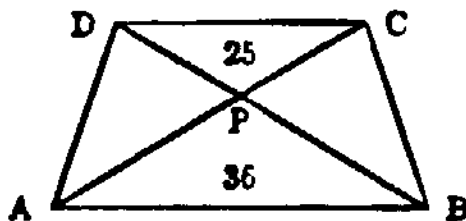


Fig 13