NEW MEXICO MATHEMATICS CONTEST XXV

FEBRUARY 1, 1992

FINAL ROUND (THREE HOURS)

Good presentation counts ! Your presentation should be clear, concise and complete.

1. Fig 1 is an example of a magic square of order 4. The sum of the four integers in each row, column, or diagonal is always 34. So we say the *magic sum* of this magic square is 34. Suppose there is a magic square of order 7, with integers from 1 through 49, what would be the magic sum ?

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Fig 1

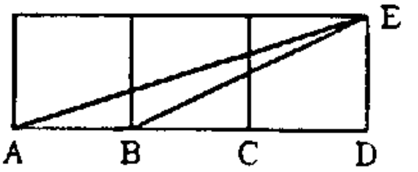

Fig 2

2. Arrange 3 squares of the same size as in Fig 2.

   Find the *sum* of the two angles $\angle DAE$ and $\angle DBE$.

3. Suppose a, b, c are the lengths of 3 sides of a triangle. If $a^2 + ab + b^2 = c^2$, then the largest angle of the triangle is 120^0. Observe that

$$\begin{aligned} 5^2 + 5 \times 3 + 3^2 &= 7^2, \\ 7^2 + 7 \times 8 + 8^2 &= 13^2, \\ 9^2 + 9 \times 15 + 15^2 &= 21^2, \\ 11^2 + 11 \times 24 + 24^2 &= 31^2, \\ 13^2 + 13 \times 35 + 35^2 &= 43^2. \end{aligned}$$

   Find positive integers x and y such that $17^2 + 17x + x^2 = y^2$.

4. In $\triangle ABC$,

$$\overline{BC} = 40\,(cm), \quad \overline{CA} = 24\,(cm), \quad \angle C = 120^0.$$

   Suppose the bisector of $\angle C$ meets the opposite side AB at D.

   (a) Find the length $\overline{AB}$.

   (b) Find the length $\overline{AD}$.

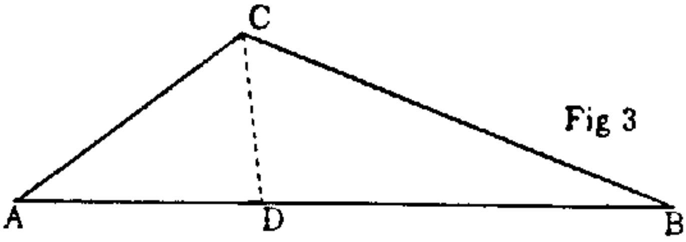

Fig 3

5. On reviewing last year's New Mexico Mathematics Contest problems, Debbie said: '*A perfect square, whose last digit is 6, must have an odd digit right before the last digit*'. Irene said: 'I was thinking the converse, *a perfect square, whose next-to-the-last digit is odd, must have 6 as the last digit*'.

Prove or disprove each of their assertions.

6. A square sheet of paper $ABCD$ is folded as shown in Fig 4 with D falling on E, which is on BC, with A falling on F, and EF intersects AB at G.

Prove that, regardless of the position of E on the side BC, the perimeter of $\triangle EBG$ is a constant length, and express this length in terms of the length l of a side of the square $ABCD$.

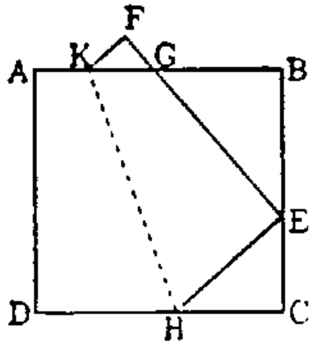

Fig 4

7. Suppose a cubic polynomial $x^3 + px^2 + qx + 72$ is divisible by both $x^2 + ax + b$ and $x^2 + bx + a$ (where a, b, p, q are constants and $a \neq b$).

Find the zeros (roots) of the cubic polynomial.

8. Suppose the diagonals AC and BD of a convex quadrangle $ABCD$ intersect at P, and the areas of $\triangle PAB$ and $\triangle PCD$ are 16 (cm^2) and 25 (cm^2), respectively.

What is the minimum possible area of the quadrangle $ABCD$?

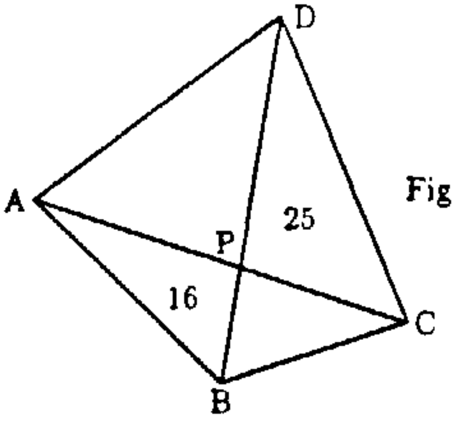

Fig 5