What do New Mexico Mathematics Teachers Need to Know?

Teachers need to have an opportunity to develop a Mathematical Knowledge for Teaching. It is only realistic to expect such knowledge to develop over years of professional study, undertaken alone, with other teachers, and in continuing education classes. However, its foundation—deep understanding of school mathematics—must be laid during pre-service education.

-MET Document

1. Introduction

From May 29 – June 1, 2007, 45 teachers and teacher educators met in Taos, New Mexico to formulate a plan to improve teacher education in the state of New Mexico. Through reading, observing teaching, and discussing what we read and observed, we developed the outline for this document. The primary readings included excerpts from “Knowing Mathematics for Teaching” by Ball, Hill and Bass, “Knowing and Teaching Elementary Mathematics” by Liping Ma, “The Mathematical Education of Teachers” from the Conference Board of the Mathematical Sciences, and “Bridging the Gap between Standards and Achievement” by Richard F. Elmore. All grade-specific recommendations come from the National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points.

This paper will ultimately be a chapter in a larger document that addresses middle school teacher preparation, high school teacher preparation, and teacher professional development in mathematics at all levels for New Mexico teachers.

2. General recommendations

The central recommendations of this document are that mathematics courses for pre-service teachers need to cover less material but go into more depth, be designed to help teachers develop the specific kind of mathematical knowledge needed to teach mathematics to others, and should provide a foundation for teachers to develop a profound understanding of the mathematics that they teach. Liping Ma describes a teacher who has a Profound Understanding of Fundamental Mathematics (PUFM) as someone who

- Knows and makes connections among different mathematical concepts and procedures.
- Appreciates the different facets of an idea and various approaches to a solution as well as their advantages and disadvantages; their mathematical knowledge is flexible.
- Knows the simple but powerful basic concepts and principles of elementary and/or middle school mathematics.
- Has a fundamental understanding of the whole elementary or middle school mathematics curriculum; their knowledge has longitudinal coherence.

While we recognize that mathematics courses must have specific content learning goals, there are certain attitudes and dispositions about mathematics teaching and learning that are equally important for teachers to develop. Since it is not possible for teachers to learn everything they
need to know about mathematics in two or three semester courses, it is critical that teachers have the skills and disposition to be life-long learners. Pre-service teachers need to leave their undergraduate training believing that math can and should make sense and that they have the ability to make sense of it for themselves. They also need to be able to communicate their ideas to others. Ultimately, they must understand that their job will be to help other people to do and make sense of mathematics.

The Mathematical Education of Teachers (MET) document has specific recommendations for the number of credit-hours in mathematics that pre-service teachers should take:

Although the quality of mathematical preparation is more important than the quantity, the following amount of mathematics coursework for prospective teachers is recommended.

(i) Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.

(ii) Prospective middle grades teachers of mathematics should be required to take at least 21 semester-hours of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers. (Note that NM requires 24 credit hours already.)

(iii) Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics, that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics.

The next section of this document focuses on the mathematics content courses for elementary school teachers. New Mexico state law currently only requires six credit-hours in mathematics for elementary certification. However, we strongly support asking the state to raise that requirement to the nine credit-hours recommended in the MET document. Thus, the content recommendations that follow specify what should be in the six credit hours currently required, and the content appropriate for a third course will be indicated as such. An outline for the additional 12 credit-hours that is appropriate for middle-grade teachers and the structure of pre-service teacher education for high school mathematic teachers is left to a future document.

The level and rigor of mathematics courses for pre-service teachers should not be lower than the level and rigor of mathematics courses required by other non-STEM majors, though the nature of the content should be specific to what teachers need. The New Mexico Mathematics Standards should be used for guidance here, both in specific content and in the processes by which mathematics is learned and taught. Instructors of pre-service teachers should model the kind of instruction that teachers are expected to use. The Process Standards in Mathematics list five guiding principles:
• **Problem Solving.** Problem solving is a tool for solidifying mathematical knowledge and learning to apply it in a variety of situations (which may not look exactly like familiar examples from a textbook), but it is also a tool for building new mathematical knowledge, and pre-service teachers need to understand and experience this. (For example, using various simple strategies to find the areas of different shapes, a class can develop the general formulas for the areas of triangles, parallelograms, etc. The formulas can be much more meaningful and memorable when developed in this way.)

• **Reasoning and Proof.** Students should be able to check and justify their answers using careful reasoning. More generally, they should understand that reasoning and proof are fundamental to the very nature of mathematics.

• **Communication.** This includes oral communication, written communication, and the effective and meaningful use of precise mathematical language. Communication is important not only for learning mathematics deeply, but also for preparing students to use mathematics as an effective tool in other disciplines.

• **Connections.** In order for students to learn that mathematics is a coherent whole and not just a long list of unrelated tricks and formulas, teachers need to understand the myriad connections which exist between different mathematical topics. (The Pythagorean Theorem, for example, involves rich connections between geometry, algebra, and number.) It is also important to make connections between mathematics and other disciplines. As with the other Process Standards, learning and understanding these connections is a lifetime pursuit for any instructor of mathematics at any level. However, laying the groundwork – understanding the importance of learning mathematics in a connected way – needs to begin for teachers in their pre-service training.

• **Representation.** This includes visual representations of numbers and operations, the number line (perhaps the most important representation for understanding high school mathematics and beyond), graphs in a coordinate plane, symbolic representations (which often support the first steps in the development of abstract mathematical thinking), and verbal descriptions, and many more. Learning to choose and use representations as effective tools in solving problems, understanding concepts, and communicating ideas is important for any student of mathematics, but it is particularly important for future teachers. Teachers need to not only be able to represent a concept in a way that is clear to themselves, they also need to be able represent a concept in a variety of ways in order to address the variety of student learning styles and backgrounds.

Finally, instructors of mathematics courses for pre-service teachers need to begin with what their students come in knowing; teachers will not have a chance to develop PUFM if those who teach them do not meet them where they are and help them move on. At the same time, instructors must cultivate those students who already have enthusiasm for mathematics; both ends of the spectrum must be supported and challenged in our courses for teachers.
3. Specific content recommendations

Because our time with pre-service teachers is limited, courses designed for them must focus on the big ideas of school mathematics. The NCTM Curriculum Focal Points identify three key learning goals for each grade level in grades K-8. Thus, these focal points can help us determine the core material that should be covered in mathematics courses for pre-service teachers.

a. NCTM Curriculum Focal Point Guidelines

i. Numbers and operations: Developing an understanding of and fluency with numbers is the core goal of K-5 mathematics. Thus, pre-service teachers need to study the following topics at a deep level:

- The structure of the rational number system. The Focal Points indicate that understanding whole number arithmetic is a core learning goal that develops through grades K-5. Furthermore, understanding of fractions and decimals is developed in grades 3-6. It is critical to point out that in addition to algorithms and number facts, children need to develop a deep and coherent understanding of rational numbers and operations.
- The place-value structure of our numeration system. The Focal Points specifically identify core learning goals related to place-value in grades 1, 2, and 4 and specifically mention it in the Connections to the Focal Points in grades 1-5.

ii. Geometry: Children must have a solid foundation in basic geometric concepts during their K-5 schooling. In order for teachers to support their students’ learning of geometry, they should study:

- Basic geometric objects and systems of measurement related to one-, two-, and three-dimensional figures. Key learning goals in grades K-6 include describing, composing and decomposing, and measuring geometric figures in one-, two-, and three-dimensions. Linear measure is a focus in grade 2, area in grade 4, and volume and surface area in grade 5.

iii. Global concepts for the elementary mathematics sequence: Because there are both mathematical concepts and dispositions about doing mathematics that will not be fully developed in a single semester course, certain mathematical themes should be treated throughout pre-service K-5 teachers’ mathematical coursework. Many of these topics are properly considered topics of the middle school curriculum, but in order for teachers’ mathematical knowledge to have longitudinal coherence, they must have lots of opportunities to think about how K-5 topics mature in higher grades. The concept of ratio is a central example of this. Children skip-count in kindergarten; it is important for kindergarten teachers to know that skip counting blooms as
multiplication and forms the roots for ratios. Since ratios are one of the core concepts of the middle school curriculum, elementary school teachers should have lots of opportunity to make connections between this concept and the other core topics of the K-5 curriculum. Global concepts that should be revisited throughout the elementary mathematics sequence include:

- **Ratios.** The NCTM focal points state that in grade 6, students should “connect ratio and rate to multiplication and division.”
- **Patterns.** Pre-service teachers should have opportunities to recognize and find rules or patterns from tables, lists and graphs.
- **Algebra.** Algebra is generalized arithmetic, and teachers who understand this will teach arithmetic in richer, deeper ways.

### b. Mathematics courses for pre-service elementary teachers

Given the above recommendations, it makes sense to lay out the two currently required mathematics courses for pre-service teachers as follows:

**Course 1:**

- **Core content:** Numbers and operations: the structure of the rational numbers; the base-ten numeration system.
- Connections of the core content to ratios and algebra.
- Opportunities for problem solving and an emphasis on mathematical representation, reasoning, and explanation throughout. Pre-service teachers need to see how mathematics can help them and their students solve real-world problems.

Learning about the rational number system is a much more subtle and complex process than many people imagine and underpins the core conceptual structure of the elementary school curriculum. An understanding of integers and integer arithmetic as well as fraction and decimal representations of rational numbers, the relationships between these representations, and different possible algorithms as well as the reasoning behind these algorithms takes many years to develop. Furthermore, procedural proficiency with rational number arithmetic is of little use if students (and teachers) do not know how to apply it in concrete situations and future mathematics courses. Pre-service teachers need to see that rational number arithmetic lays the foundations for students to understand ratios and proportions as well as algebra (as generalized arithmetic). For example, teachers should see the connections between skip-counting, unit pricing, ratios and rates, and linear equations.
Course 2:

- **Core content:** Geometry and measurement.
- Connections of the core content to the rational number system, ratios, and algebra.
- Opportunities for problem solving and an emphasis on mathematical representation, reasoning, and explanation throughout. Pre-service teachers need to see how mathematics can help them and their students solve real-world problems.

The idea of a unit is one of the unifying themes in K-8 mathematics:

![Units concept](image)

2 apples, 2 tenths, 2 thirds, 2 centimeters, 2 square inches, 2 reference segments on the number line. Units finally get an explicit treatment when children study measurement, where they encounter both culturally defined systems of measurement (such as the metric system) and the more abstract notions of measurement that are embodied by length, area, and volume. Much of the geometry of the K-8 grades (that isn’t merely descriptive) structures and supports students’ understanding of length, area and volume. For example, 2-D transformations, symmetry of plane figures, and tilings of the plane all relate to a developing conception of area. Furthermore, connections between arithmetic and geometry are critical. For example, the area of a rectangle is one of the most important representations of multiplication as it is the only common representation that generalizes to irrational numbers, and coordinate geometry (even at a very basic level) is one of the most powerful tools for studying algebraic concepts.

**A third course for elementary teachers:**

While both ratios and algebraic concepts and use of symbolic notation should be incorporated throughout the first two courses as “global” topics, it needs to be made a specific focus in the third course. Furthermore, while probability and data analysis are not indicated as one of the core focal points until grade 8, they are mentioned in every “Connections to the Focal Points” and are included in the New Mexico state standards starting in kindergarten. Thus, a third mathematics course for elementary teachers should also address probability and data. This is especially appropriate because ratios should have a greater emphasis in this course as well, and connections between ratios and probability are natural.
Institutions of Higher Education (IHEs) who teach pre-service teachers are strongly encouraged to work together to develop assessments for these courses to be sure that their students are developing the kind of deep mathematical knowledge needed to teach elementary school mathematics. Ideally, such assessments will measure not just content knowledge, but mathematical knowledge for teaching.

4. The Broader Context

Mathematics content courses for pre-service teachers are part of the teacher professional continuum, which starts with a teacher’s own K-12 schooling and continues into methods courses and licensure programs as well as classroom experience and in-service professional development. To be effective, instructors of mathematics content courses must work with others who are part of this professional continuum.

First, instructors must work together with others who teach these same courses, both within their own departments and across institutions of higher education, to assure consistency and to share best practices. Specifically, we recommend that faculty who teach preservice mathematics content work to develop assessment items for common courses and organize and attend regular meetings across institutions.

Second, faculty who teach courses for pre-service elementary teachers will be most effective when they work with education departments where pre-service teachers explicitly study other aspects of mathematics learning, such as the development of mathematical thinking in children, strategies for reducing math phobia, the contexts of children's lives that impact their learning opportunities, and so on. In particular, instructors should strive to coordinate the math content courses with the math methods courses and other opportunities to learn about instruction and classrooms. Pre-service teachers need to see examples of good classroom teaching for kids that come from good curricula. Examples of teaching that they see should be relevant, for instance from a research-based curriculum that is used in the local school system. One way for pre-service teachers to have these kinds of experiences early in the education is to require a one-hour lab to go with content courses that requires time in K-8 classrooms. Another is to link content and methods course, require co-requisites, or offer courses that are collaboratively taught. Instructors of content and methods courses need to work together to find ways to expose pre-service teachers to the New Mexico Mathematics Standards—especially the process standards—and need to be aware of the kinds of mathematical professional development opportunities that exist for them after they enter classrooms.

Finally, instructors should work with in-service teachers and school systems. Because content courses for pre-service teachers are a small—but critical—part of the teacher professional continuum, to be most effective they must be informed by what teachers need in real classrooms. In-service professional development in mathematics needs to connect to and build on teachers’
pre-service educational experiences, and support for learning must continue throughout teachers’ careers.

In order for collaboration between teachers and instructors of pre-service content courses to successfully implement these recommendations, they will need extensive support from their institution’s administrations. The work of collaboration between instructors, between different departments and colleges, and between universities and school systems needs to be recognized and rewarded.

References

“Knowing Mathematics for Teaching” by Ball, Hill and Bass,

“Knowing and Teaching Elementary Mathematics” by Liping Ma,

“The Mathematical Education of Teachers” from the Conference Board of the Mathematical Sciences

“Bridging the Gap between Standards and Achievement” by Richard F. Elmore.

NCTM Focal Points