



SYLLABUS FOR ALGEBRA QUALIFYING EXAM

Topics

1. Group Theory

- (a) Basic facts such as homomorphisms, subgroups and cosets, normal subgroups, quotients, automorphisms, short exact sequences and extensions.
- (b) Symmetric and Alternating groups.
- (c) Free groups.
- (d) Abelian groups, structure theorem for finitely generated abelian groups.
- (e) Actions of groups on sets.
- (f) Sylow theorems.
- (g) Nilpotent, solvable, normal and subnormal series.
- (h) Krull-Schmidt Theorem.

2. Ring Theory

- (a) Basic facts such as homomorphisms and subrings.
- (b) Ideal theory.
- (c) Factorization in commutative rings: UFD's, PID's and Euclidean domains.
- (d) Polynomial rings.
- (e) Chain conditions.

3. Module Theory

- (a) Basic facts such as homomorphisms, submodules, exact sequences, free modules, torsion modules.
- (b) The structure theorem for finitely generated modules over PIDs.
- (c) Projective and injective modules.
- (d) Tensor products, Hom, Tor, and Ext.
- (e) Noetherian modules, chain conditions.

4. Linear Algebra

- (a) Basic facts such as vector spaces, linear transformations, determinants.
- (b) Characteristic polynomials, eigenvectors and eigenvalues.
- (c) Canonical forms.

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5. Field Theory

- (a) Algebraic and transcendental field extensions, splitting fields, algebraic closure, normality, separability.
- (b) Galois theory.
- (c) Finite fields.
- (d) Cyclic and cyclotomic extensions.

Suggested References

1. Algebra by Hungerford, Springer-Verlag.
2. Basic Algebra I by Jacobson, Freeman.
3. Topics in Algebra by Herstein, Ginn-Blaisdell.
4. Algebra by Goldhaber and Ehrlich, MacMillan.
5. Algebra by Lang, Addison Wesley.
6. Modern Algebra by van der Waerden, Ungar.
7. The Theory of Groups by Rotman, Allyn and Bacon.
8. The Theory of Groups by Hall, MacMillan.
9. The Theory of Rings by McCoy, MacMillan.
10. Steps in Commutative Algebra by Sharp, Cambridge.
11. Galois Theory by Rotman, Springer-Verlag.
12. Classical Galois Theory by Gaal, Chelsea.