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Tenth New Mexico Analysis Seminar

Department of Mathematics and Statistics
University of New Mexico
Albuquerque
October 11-12, 2007

ABSTRACTS

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Minicourse A: *Bilinear Operators in Analysis and PDEs.*
Main Lecturer: Andrea Nahmod, University of Massachusetts, Amherst, MA.
Guest Speaker: Rodolfo Torres, University of Kansas, Lawrence, KA.

- *Part I: Translation invariant smooth and nonsmooth bilinear multipliers.*
  Speaker: Andrea Nahmod.
- *Part II: Bilinear pseudo-differential operators: beyond Coifman-Meyer's theory.*
  Speaker: Rodolfo Torres.
- *Part III: The T(1,1)-theorem for modulation invariant bilinear singular operators.*
  Speaker: Andrea Nahmod.

Abstract: Bilinear and more general multilinear estimates have for a long time played an ubiquitous and essential role in harmonic analysis and linear and nonlinear PDEs. Pioneer and fundamental work by J.M. Bony and by R. Coifman and Y. Meyer has been and still is central to important problems in these areas. Still, questions remain as to the study of operators which could have singular multipliers or symbols with 'non-standard' decay conditions; and other nonlinear transformations. In this aspect, the field is still open and challenging.

In these lectures, we will attempt to describe work done in recent years by many people in the subject and discuss some of the open questions.

The starting point in the first lecture will be a discussion of bilinear operators with non-smooth multipliers and of a comprehensive criterion in one dimensions ensuring their boundedness. We will describe the wave packet or time-frequency analysis of the problem; pioneered by C. Fefferman in his proof of Carleson's theorem on the *a.e.* convergence of Fourier series of L^2-functions, and further developed by Lacey and Thiele in their celebrated proof of A. P. Calderón's question on the *bilinear Hilbert transform.* The second lecture will be devoted to the unfolding theory of multilinear pseudo-differential operators with x-dependent symbols. Some new results for bilinear pseudo-differential operators beyond the results available for the so called Coifman-Meyer class will be explained. We will describe in particular how the bilinear pseudo-differential setup differs from the linear pseudo-differential one in terms of symbolic calculus and boundedness properties on products of Lebesgue and/or Sobolev spaces. A new criterion for boundedness of modulation invariant bilinear pseudo-differential operators; namely a $T(1,1)$-theorem will be presented. The third lecture will be devoted to the study and proof of such class as well as to the description of some other open problems and unanswered questions.

Minicourse B: *Martingales and Fourier multipliers. What is new in this old marriage?*
Main Lecturer: Rodrigo Bañuelos (Purdue University, West Lafayette, IN).
Guest Speaker: Oliver Dragičević (University of Ljubljana, Slovenia).

- *Part I and II: Martingales and Fourier multipliers. What is new in this old marriage?*
  Speaker: Rodrigo Bañuelos.

- *Part III: Martingales and Fourier multipliers. What is new in this old marriage?*
  Speaker: Oliver Dragičević.

Abstract: Many classical singular integral operators, like the Hilbert transform on the real line, the Riesz transforms on $\mathbb{R}^n$ and the Beurling-Ahlfors singular integral operator in the plane, can be represented as certain stochastic integrals with respect to Brownian motion. As it turns out, this class of Fourier multipliers can be expanded considerably by allowing stochastic integrals relative to Lévy processes. The goal of the two lectures by Rodrigo Banuelos is to show how these representations and the celebrated sharp inequalities for martingales due to Burkholder can lead to nearly optimal bounds on L^p constants for these operators.

A different approach to some of these bouonds that does not use stochastic integration but still appeals to the Burkholder martingale inequalities was developed by Nazarov and Volberg using Bellman functions. Yet another application of the Burkholder's inequality goes through representation of the Ahlfors-Beurling operator as an average of martingale transforms. The lecture by Oliver Dragičević will explain these connections.

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Short Talks

- Scot Childress (University of California, Riverside)
  Title: *Canonical products over the roots of certain Dirichlet polynomials.*
  Abstract: We define a (real) Dirichlet polynomial P of a complex variable s as an expression $P(s) = m_0 r_0^s + \cdots + m_M r_M^s$, where $0 \neq m_j \in \mathbb{C}$ and $0 < r_M < \cdots < r_0$ are real numbers. Associated to a (real) Dirichlet polynomial P is its canonical product, G_P, given by $G_P(s) = s^h \prod_{P(\omega)=0} \left(\frac{s}{w}\right) e^{s/w}$ (h denotes the multiplicity of 0 as a root of P). We use the Diophantine approximation scheme and the *lattice/nonlattice* dichotomy, expaunded in *Fractal Geometry and Complex Dimensions* (Lapidus, Van Frankenhuysen), to show that G_P defines an entire function and that if $Q_n \to P$ is a lattice approximation of P then, with some minor adjustment, $G_{Q_n} \to G_P$ uniformly on compact sets. Classical techniques in the theory of infinite products are employed to derive the factorization formula $P(s) = K_P e^{K_P s} G_P(s)$ for lattice polynomials.

We apply the obtained canonical product convergence results to recover the same factorization formula for nonlattice polynomials.

- Hung Lu (University of California, Riverside)
  Title: *Nonarchimedean Cantor Set and String.*
  Abstract: We construct a nonarchimidean (or $p - adic$) analogue of the classical ternary Cantor set $\mathcal{C}$. In particular, we show that this nonarchimedean Cantor set $\mathcal{C}_3$ is self-similar. Furthermore, we characterize $\mathcal{C}_3$ as the subset of 3-adic integers whose elements contain only 0's and 2's in their 3-adic expansions and prove that $\mathcal{C}_3$ is naturally homeomorphic to $\mathcal{C}$. Finally, from the point of view of the theory of fractal strings and their complex fractal dimensions (Lapidus, Van Frankenhuysen), the corresponding nonarchimedean Cantor string resembles the standard real (or *archimedean*) cantor string perfectly.

- Terry Loring (University of New Mexico, Albuquerque)
  Title: *Renyi dimension and Gaussian filtering*
  Abstract: We will discuss rigourously how the generalized fractal dimensions of a measure, such as the correlation dimension, can be determined by convolving that measure against a semigroup of Gaussians. This is for finite, compactly supported Borel measures on finite dimensional Euclidean space. The L^p norms for the resulting functions decay at different rates that depend on the generalized fractal dimensions of the measure. We will discuss applications in image anaylsis, especially related to Laplacian pyramids.

- Jorge Noriega-Rivera (Universidad Autónoma del Estado de Morelos, México)
  Title: *A parabolic version of singular integrals on manifolds.*
  Abstract: We introduce a class of parabolic uniformly rectifiable sets on which certain parabolic singular integral operators are L^p bounded.

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Organizers: Tiziana Giorgi (NMSU), Joseph Lakey (NMSU),
Cristina Pereyra (UNM), Adam Sikora (NMSU), Robert Smits (NMSU).