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Maximum norm estimates for the solutions of the incompressible Navier-Stokes equations

Consider the Cauchy problem for the incompressible Navier-Stokes equations. Assume that there is a bound for the initial data $\mathbf{u}(x,0)$ in the maximum norm, i.e., $|\mathbf{u}(x,0)|_\infty < \infty$. We want to prove that there is a unique solution $\mathbf{u}(x,t)$ which for $t > 0$ is smooth and exists in some time interval $0 < t < T$. We can estimate T and the derivatives of $\mathbf{u}$ in terms of $|\mathbf{u}(x,0)|_\infty$. The estimates do not depend on the L_2-norm $\|\mathbf{u}(x,0)\|$. However, if $\|\mathbf{u}(x,0)\| < \infty$, then we can localize the estimate in space and compute the local smallest scale.