HOMEWORK 6

An Example

You often must solve two linear systems where the coeffients of the variables are the same. You can solve these by "twice augmenting" the coefficient matrix.

Example. Find the solution for each sets of linear equations. Do so by row reducing a single matrix to row echelon form and then back-soving separately.

$$
\begin{array}{rrrrr}
x_1 & - & 2x_2 & - & x_3 & = & -2 \\
2x_1 & - & 4x_2 & - & 3x_3 & = & -7 \\
x_1 & - & 2x_2 & + & x_3 & = & 4
\end{array}
\qquad
\begin{array}{rrrrr}
x_1 & - & 2x_2 & - & x_3 & = & 2 \\
2x_1 & - & 4x_2 & - & 3x_3 & = & 5 \\
x_1 & - & 2x_2 & + & x_3 & = & 0
\end{array}
$$

Example 1. The coeffient matrix in both cases is

$$
\begin{bmatrix}
1 & -2 & -1 \\
2 & -4 & -3 \\
1 & -2 & 1
\end{bmatrix}
$$

and we will augment by both of the "right sides." This gives us

$$
\begin{bmatrix}
1 & -2 & -1 & -2 & 2 \\
2 & -4 & -3 & -7 & 5 \\
1 & -2 & 1 & 4 & 0
\end{bmatrix}.
$$

So we do the row operations until we hit row echelon form:

$$
\begin{bmatrix}
1 & -2 & -1 & -2 & 2 \\
2 & -4 & -3 & -7 & 5 \\
1 & -2 & 1 & 4 & 0
\end{bmatrix}
\qquad
\begin{bmatrix}
1 & -2 & -1 & -2 & 2 \\
0 & 0 & 1 & 3 & -1 \\
0 & 0 & 2 & 6 & -2
\end{bmatrix}
$$
$$
\downarrow \qquad\qquad\qquad\qquad \downarrow
$$
$$
\begin{bmatrix}
1 & -2 & -1 & -2 & 2 \\
0 & 0 & -1 & -3 & 1 \\
0 & 0 & 2 & 6 & -2
\end{bmatrix}
\qquad
\begin{bmatrix}
1 & -2 & -1 & -2 & 2 \\
0 & 0 & 1 & 3 & -1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$
$$
\downarrow
$$

This converts back to equations as

$$
\begin{array}{rrrrr}
x_1 & - & 2x_2 & - & x_3 & = & -2 \\
& & & & x_3 & = & 3 \\
& & & & 0 & = & 0
\end{array}
\qquad
\begin{array}{rrrrr}
x_1 & - & 2x_2 & - & x_3 & = & 2 \\
& & & & x_3 & = & -1 \\
& & & & 0 & = & 0
\end{array}
$$

For the first, we back solve

$$
\begin{aligned}
x_1 &= -2 + 2x_2 + 3 \\
x_3 &= 3
\end{aligned}
$$

so the solution is

$$
\begin{aligned}
x_1 &= 1 + 2r \\
x_2 &= r \qquad (\text{any } r) \\
x_3 &= 3
\end{aligned}
$$

For the second, we find

$$
\begin{aligned}
x_1 &= 2 + 2x_2 + (-1) \\
x_3 &= -1
\end{aligned}
$$

so the solution is

$$
\begin{aligned}
x_1 &= 1 + 2r \\
x_2 &= r \qquad (\text{any } r) \\
x_3 &= -1
\end{aligned}
$$