REVIEW III-KEY

1.

Problem 1. Let

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 1 & 2 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 & -3 & 0 \end{bmatrix}.$$

Find a basis for

$$\text{col}(A^T),\ \text{col}(A),\ \text{null}(A)$$

and find

$$\text{rank}(A)$$

and

$$\det(A),\ A^{-1}$$

if they exist.

2.

Problem 2. Find the solution set S to the equations

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot \mathbf{x} = \mathbf{0}$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot \mathbf{x} = \mathbf{0}.$$

Is S a subspace of $\mathbb{R}^4$? If so, what is its dimension?

3.

Problem 3. Suppose $\mathbf{v}$ is an eigenvector for the eigenvalue 2 of the matrix A. What is

$$(A^4 + A^3 + A^2 + A + I)\mathbf{v}?$$

4.

Problem 4. Consider V, the vector space of all strictly-upper triangular matrices:

$$V = \left\{ \begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix} \middle| a, b, c \in \mathbb{R} \right\}.$$

Let

$$X = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Show that if $A \in V$ then $XA \in V$.

5.

Problem 5. Determine the rank, nullity and determinant of each of the following:

(a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 8 & 13 \\ 3 & 5 \end{bmatrix}$$

6.

Problem 6. Show that 2 is *not* an eigenvalue of

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

7.

Problem 7. Let

$$A = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix}.$$

Find X and D so that D is diagonal and

$$A = XDX^{-1}.$$

8.

Problem 8. Find all r so that

$$A = \begin{bmatrix} 2-r & 1 & r & 1 \\ 1 & 2 & r & 1 \\ 2 & 2 & 1+2r & 1 \\ 1+r^2 & 2 & r & 1+r^2 \end{bmatrix}$$

is invertible.

9.

Problem 9. Let

$$\mathbf{x} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

and

$$\mathbf{a} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

and

$$\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

(a) Find the vector projection of $\mathbf{x}$ onto $\mathbf{a}$.
(b) Find the vector projection of $\mathbf{x}$ onto $\mathbf{b}$.
(c) Find the vector projection of $\mathbf{a}$ onto $\mathbf{b}$.
(d) Find the closest vector to $\mathbf{x}$ that is in the span of $\mathbf{a}$ and $\mathbf{b}$.

10.

Problem 10. Define

$$T : \mathbb{R}^3 \to \mathbb{R}^2$$

by

$$T(\mathbf{x}) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

What is the matrix representation of T with respect to $\mathcal{B} - \mathcal{C}$ where

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

and

$$\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}.$$